Finite Population Distribution Function Estimation Using Auxiliary Information Under Simple Random Sampling

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Abstract

In this paper, a new estimator for estimating the finite population distribution function (DF) are propose using supplementary information on the DF of the auxiliary variable under simple random sampling. A comparative study is conducted to compare, theoretically and numerically, the adapted distribution function estimators of Cochran (1940), Murthy (1967), Bahl and Tuteja (1991), Rao (1991), Singh et al. (2009) and Grover and Kaur (2014) with the proposed estimators. It is found that the proposed estimators always perform better than the adapted estimators in terms of MSE and percentage relative efficiency.

Keywords: Distribution Function, Estimator, Sampling distribution, Statistics.

Introduction

In the theory of survey sampling, suitable use of the auxiliary information plays an important role in increasing the precision of an estimator of the unknown population parameter(s). A lot of ratio, product and regression type estimators of the population mean have been proposed in the literature of survey sampling. Ratio estimators are widely used to estimate the population mean when there exists a high correlation between the study variable and the auxiliary variable. Several authors including Murthy (1967), Srivastava and Jhaji (1983), Rao (1991), Upadhyaya and Singh (1999), Singh (2003), Shabbir and Gupta (2005), Kadilar and Cingi (2005), Kadilar and Cingi (2006) Gupta and Shabbir (2008), Singh et al. (2009) Grover and Kaur (2011) and Grover and Kaur (2014), have devised different types of estimators that utilize supplementary information in terms of one or more auxiliary variables to estimate the population mean.

The rest of paper is organized as follows. In Section 2, some notations are given. In Section 3, we adapt some estimators of finite population mean for the finite (DF). The proposed estimators are given in Section 4. In Section 5, numerical comparisons are performed. And 6 concern with empirical studies, finally, conclusions are drawn in Section 7.

Notations

Consider a finite population $\Omega = \{1, 2, \dots, N\}$ of N distinct units. In order to estimate the finite population distribution function, a sample of size n units is drawn from this population using simple random sampling without replacement. Let

Y : the study variable,

X : the auxiliary variable,

Z : ranks of X ,

$I(Y \leq t_y)$: indicator variable based on Y ,

$I(X \leq t_x)$: indicator variable based on X ,

where $x = \tilde{X}$ and $y = \tilde{Y}$. Here, $\tilde{X}$ and $\tilde{Y}$ is the population medians of X and Y

$F(t_y) = \sum_{i=1}^N I(Y_i \leq t_y)/N$: the population distribution function of Y ,

$\hat{F}(t_y) = \sum_{i=1}^n I(Y_i \leq t_y)/n$: the sample distribution function of Y ,

$F(t_x) = \sum_{i=1}^N I(X_i \leq t_x)/N$: the population distribution function of X ,

$\hat{F}(t_x) = \sum_{i=1}^n I(X_i \leq t_x)/n$: the sample distribution function of X ,

$S_{F_{t_y}}^2 = \sum_{i=1}^N (I(Y_i \leq t_y) - F(t_y))^2 / (N - 1)$: the population variance of $I(Y \leq t_y)$,

$S_{F_{t_x}}^2 = \sum_{i=1}^N (I(X_i \leq t_x) - F(t_x))^2 / (N - 1)$: the population variance of $I(X \leq t_x)$,

$C_{F_{t_y}} = S_{F_{t_y}} / F(t_y)$: the population coefficient of variation of $I(Y \leq t_y)$,

$C_{F_{t_x}} = S_{F_{t_x}} / F(t_x)$: the population coefficient of variation of $I(X \leq t_x)$,

$S_{F_{t_y}F_{t_x}} = \sum_{i=1}^N \{(I(Y_i \leq t_y) - F(t_y))(I(X_i \leq t_x) - F(t_x))\} / (N - 1)$: the population covariance between $I(Y \leq t_y)$ and $I(X \leq t_x)$,

$R_{F_{t_y}F_{t_x}} = S_{F_{t_y}F_{t_x}} / (S_{F_{t_y}} S_{F_{t_x}})$: the population correlation coefficient between $I(Y \leq t_y)$ and $I(X \leq t_x)$,

In order to obtain the biases and mean squared errors (MSEs) of the existing and proposed estimators of $F(t_y)$, we consider the following relative error terms. Let

$$e_0 = \frac{\hat{F}(t_y) - F(t_y)}{F(t_y)} \text{ and } e_1 = \frac{\hat{F}(t_x) - F(t_x)}{F(t_x)},$$

such that $e_i = 0$ for $i = 0, 1$, where $E(\cdot)$ is the mathematical expectation of $(\cdot)$.

$$E(e_0^2) = \lambda C_{F_{t_y}}^2 = \varphi_{20}, E(e_1^2) = \lambda C_{F_{t_x}}^2 = \varphi_{02}, E(e_0 e_1) = \lambda R_{F_{t_y}F_{t_x}} C_{F_{t_y}} C_{F_{t_x}} = \varphi_{11}.$$

Adapted estimators in simple random sampling

In this section, some estimators of finite population mean are adapted for estimating the finite CDF under simple random sampling, which are exist for mean estimation. The biases and MSEs of these adapted estimators are derived under the first order of approximation.

1. The traditional unbiased estimator of $F(t_y)$ is

$$\hat{F}(t_y) = \frac{1}{n} \sum_{i=1}^n I(Y_i \leq t_y). \quad (1)$$

The variance of $\hat{F}(t_y)$ is

$$\text{Var}(\hat{F}(t_y)) = F^2(t_y)\varphi_{20}. \quad (2)$$

2. Cochran (1940) existing ratio estimator of $F(t_y)$ is

$$\hat{F}_R(t_y) = \hat{F}(t_y) \left(\frac{F(t_x)}{\hat{F}(t_x)} \right). \quad (3)$$

The bias and MSE of $\hat{F}_R(t_y)$, to the first order of approximation, are

$$\begin{aligned} \text{Bias}(\hat{F}_R(t_y)) &\cong F(t_y)(\varphi_{02} - \varphi_{11}), \\ \text{MSE}(\hat{F}_R(t_y)) &\cong F^2(t_y)(\varphi_{20} + \varphi_{02} - 2\varphi_{11}). \end{aligned} \quad (4)$$

3. Murthy (1964) existing product estimator of $F(t_y)$ is

$$\hat{F}_P(t_y) = \hat{F}(t_y) \left(\frac{\hat{F}(t_x)}{F(t_x)} \right). \quad (5)$$

The bias and MSE of $\hat{F}_P(t_y)$, to the first order of approximation, are

$$\begin{aligned} \text{Bias}(\hat{F}_P(t_y)) &= F(t_y)\varphi_{11}, \\ \text{MSE}(\hat{F}_P(t_y)) &\cong F^2(t_y)(\varphi_{20} + \varphi_{02} + 2\varphi_{11}) \end{aligned} \quad (6)$$

4. Following Bahl and Tuteja (1991), the combined ratio-type and product-type exponential estimators of $F(t_y)$, respectively, are

$$\hat{F}_{BT,R}(t_y) = \hat{F}(t_y) \exp \left(\frac{F(t_x) - \hat{F}(t_x)}{\hat{F}(t_x) + F(t_x)} \right), \quad (7)$$

$$\hat{F}_{BT,P}(t_y) = \hat{F}(t_y) \exp \left(\frac{\hat{F}(t_x) - F(t_x)}{\hat{F}(t_x) + F(t_x)} \right). \quad (8)$$

The biases and MSEs of $\hat{F}_{BT,R}(t_y)$ and $\hat{F}_{BT,P}(t_y)$, to the first order of approximation, are

$$\begin{aligned} \text{Bias}(\hat{F}_{BT,R}(t_y)) &\cong F(t_y) \left(\frac{3}{8} \varphi_{020} - \frac{1}{2} \varphi_{110} \right), \\ \text{MSE}(\hat{F}_{BT,R}(t_y)) &\cong \frac{F^2(t_y)}{4} (4\varphi_{200} + \varphi_{020} - 4\varphi_{110}), \end{aligned} \quad (9)$$

and

$$\begin{aligned} \text{Bias}(\hat{F}_{BT,P}(t_y)) &\cong F(t_y) \left(\frac{1}{2} \varphi_{110} - \frac{1}{8} \varphi_{020} \right), \\ \text{MSE}(\hat{F}_{BT,P}(t_y)) &\cong \frac{F^2(t_y)}{4} (4\varphi_{200} + \varphi_{020} + 4\varphi_{110}), \end{aligned} \quad (10)$$

respectively.

5. The existing regression estimator of $F(t_y)$ is

$$\hat{F}_{Reg}(t_y) = \hat{F}(t_y) + m(F(t_x) - \hat{F}(t_x)), \quad (11)$$

where m is an unknown constant. Here, $\hat{F}_{Reg}(t_y)$ is an unbiased estimator of $\hat{F}(t_y)$. The minimum variance of $\hat{F}_{Reg}(t_y)$ at the optimum value $m_{(opt)} = (F(t_y)\varphi_{11})/(F(t_x)\varphi_{02})$ is

$$\text{Var}_{\min}(\hat{F}_{Reg}(t_y)) = \frac{F^2(t_y)(\varphi_{20}\varphi_{02} - \varphi_{11}^2)}{\varphi_{02}}. \quad (12)$$

Here, (12) may be written as

$$\text{Var}_{\min}(\hat{F}_{Reg}(t_y)) = F^2(t_y)\varphi_{20}(1 - \rho_{12}^2). \quad (13)$$

6. Rao (1991) existing difference-type estimator of $F(t_y)$ is

$$\hat{F}_{R,D}(t_y) = m_1\hat{F}(t_y) + m_2(F(t_x) - \hat{F}(t_x)), \quad (14)$$

where m_1 and m_2 are unknown constants. The bias and MSE of $\hat{F}_{R,D}(t_y)$, to the first order of approximation, are

$$\begin{aligned} \text{Bias}(\hat{F}_{R,D}(t_y)) &= F(t_y)(m_1 - 1), \\ \text{MSE}(\hat{F}_{R,D}(t_y)) &\cong F^2(t_y) - 2m_1F^2(t_y) + m_1^2F^2(t_y) + m_1^2F^2(t_y)\varphi_{20} \\ &\quad - 2m_1m_2F(t_y)F(t_x)\varphi_{11} + m_2^2F^2(t_x)\varphi_{02}. \end{aligned} \quad (15)$$

The optimum values of m_1 and m_2 , determined by minimizing (15), are

$$\begin{aligned} m_{1(\text{opt})} &= \frac{\varphi_{02}}{(\varphi_{02}\varphi_{20} - \varphi_{11}^2 + \varphi_{02})}, \\ m_{2(\text{opt})} &= \frac{F(t_y)\varphi_{11}}{F(t_x)(\varphi_{20}\varphi_{02} - \varphi_{11}^2 + \varphi_{02})}. \end{aligned}$$

The minimum MSE of $\hat{F}_{R,D}(t_y)$ at the optimum values of m_1 and m_2 is

$$\text{MSE}_{\min}(\hat{F}_{R,D}(t_y)) = \frac{F^2(t_y)(\varphi_{20}\varphi_{02} - \varphi_{11}^2)}{(\varphi_{20}\varphi_{02} - \varphi_{11}^2 + \varphi_{02})}. \quad (16)$$

Here, (16) may be written as

$$\text{MSE}_{\min}(\hat{F}_{R,D}(t_y)) = \frac{F^2(t_y)\varphi_{20}(1 - \rho_{12}^2)}{1 + \varphi_{20}(1 - \rho_{12}^2)}. \quad (17)$$

7. Singh et al. (2009) adapted generalized ratio-type exponential estimator of $F(t_y)$ is

$$\hat{F}_S(t_y) = \hat{F}(t_y) \exp\left(\frac{a(F(t_x) - \hat{F}(t_x))}{a(F(t_x) + \hat{F}(t_x)) + 2b}\right), \quad (18)$$

where $a = 1$ and $b = 0$ are known constants. The bias and MSE of $\hat{F}_S(t_y)$, to the first order of approximation, are

$$\begin{aligned} \text{Bias}(\hat{F}_S(t_y)) &\cong F(t_y) \left(\frac{3}{8}\theta^2\varphi_{02} - \frac{1}{2}\theta\varphi_{11} \right), \\ \text{MSE}(\hat{F}_S(t_y)) &\cong \frac{F^2(t_y)}{4} (4\varphi_{20} + \theta^2\varphi_{02} - 4\theta\varphi_{11}), \end{aligned} \quad (19)$$

where $\theta = aF(t_x)/(aF(t_x) + b)$. Here the value of θ become 1

8. Grover and Kaur (2014) adapted generalized class of ratio-type exponential estimator of $F(t_y)$ is

$$\hat{F}_{G,K}(t_y) = \{m_3\hat{F}(t_y) + m_4(F(t_x) - \hat{F}(t_x))\} \exp\left(\frac{a(F(t_x) - \hat{F}(t_x))}{a(F(t_x) + \hat{F}(t_x)) + 2b}\right), \quad (20)$$

where m_3 and m_4 are unknown constants. The bias and MSE of $\hat{F}_{G,K}(t_y)$, to the first order of approximation, are

$$\begin{aligned} \text{Bias}(\hat{F}_{G,K}(t_y)) &\cong F(t_y)(m_3 - 1) + \frac{3}{8}\theta^2m_3F(t_y) + \frac{1}{2}\theta m_4F(t_x)\varphi_{02} - \frac{1}{2}\theta F(t_y)\varphi_{11}, \\ \text{MSE}(\hat{F}_{G,K}(t_y)) &\cong m_4^2F^2(t_x)\varphi_{02} + m_3^2F^2(t_y)\varphi_{20} + 2\theta m_3m_4F(t_y)F(t_x)\varphi_{02} \\ &\quad - 2m_3m_4F(t_y)F(t_x)\varphi_{11} + F^2(t_y) - 2m_3F^2(t_y) + \theta m_3^2F^2(t_y) \\ &\quad + m_3F^2(t_y)\varphi_{11} - \theta m_4F(t_y)F(t_x)\varphi_{02} - 2\theta m_3^2F^2(t_y)\varphi_{11} \\ &\quad - \frac{3}{4}\theta^2m_3F^2(t_y)\varphi_{02} + \theta^2m_3^2F^2(t_y)\varphi_{02}. \end{aligned} \quad (21)$$

The optimum values of m_3 and m_4 , determined by minimizing (21), are

$$k_{3(\text{opt})} = \frac{\varphi_{02}(\theta^2\varphi_{02} - 8)}{8(-\varphi_{20}\varphi_{02} + \varphi_{11}^2 - \varphi_{02})},$$

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$$k_{4(\text{opt})} = \frac{F(t_y)(\theta^3 \varphi_{02}^2 - \theta^2 \varphi_{02} \varphi_{11} + 4\theta \varphi_{20} \varphi_{02} - 4\theta \varphi_{11}^2 - 4\theta \varphi_{02} + 8\varphi_{11})}{8F(t_x)(\varphi_{20} \varphi_{02} - \varphi_{11}^2 + \varphi_{02})}.$$

The simplified minimum MSE of $\hat{F}_{G,K}(t_y)$ at the optimum values of m_3 and m_4 is

$$\text{MSE}_{\min}(\hat{F}_{G,K}(t_y)) \cong \frac{F^2(t_y)}{64} \left(64 - 16\theta^2 \varphi_{02} - \frac{\varphi_{02}(-8 + \theta^2 \varphi_{02})^2}{\varphi_{02}(1 + \varphi_{20}) - \varphi_{11}^2} \right). \quad (22)$$

Proposed estimator in simple random sampling

On the lines of $\hat{F}_{R,D}(t_y)$, $\hat{F}_S(t_y)$ and average of $\hat{F}_{BT,R}(t_y)$ and $\hat{F}_{BT,P}(t_y)$, a new estimator is proposed for estimating $F(t_y)$ is given by

$$\hat{F}_{Prop}(t_y) = \left[\frac{\hat{F}(t_y)}{2} \left\{ \exp\left(\frac{F(t_x) - \hat{F}(t_x)}{\hat{F}(t_x) + F(t_x)}\right) + \exp\left(\frac{\hat{F}(t_x) - F(t_x)}{\hat{F}(t_x) + F(t_x)}\right) \right\} \right] \exp\left(\frac{F(t_x) - \hat{F}(t_x)}{F(t_x) + \hat{F}(t_x)}\right) + m_5(F(t_x) - \hat{F}(t_x)) + m_6 \hat{F}(t_y)$$

where m_5 and m_6 , are determined late. The estimator $\hat{F}_{Prop}(t_y)$ can also be written as

$$\hat{F}_{Prop}(t_y) = \left\{ F(t_y)(1 + e_0)(1 + m_6) - m_5 e_1 F(t_x) + \frac{1}{8} F(t_y) \right\} \left(1 - \frac{1}{2} e_1 + \frac{3}{8} \xi_1^2 + \dots \right). \quad (23)$$

Simplifying (23) and keeping terms only up to the second power of ξ 's, we can write

$$\begin{aligned} (\hat{F}_{Prop}(t_y) - F(t_y)) &\cong m_6 F(t_y) + F(t_y) e_0 + m_6 F(t_y) e_0 - \frac{1}{2} F(t_y) e_1 + \frac{1}{2} m_6 F(t_y) e_1 \\ &\quad - \frac{1}{2} F(t_y) e_0 e_1 + m_5 F(t_y) e_1^2 + \frac{1}{2} m_5 F(t_x) e_1^2 - \frac{1}{2} m_6 F(t_y) e_0 e_1 \\ &\quad + \frac{3}{8} m_6 F(t_y) e_1^2 - m_5 F(t_x) e_1. \end{aligned} \quad (24)$$

The bias and MSE of $\hat{F}_{Prop}(t_y)$, to the first order of approximation, respectively, are

$$\begin{aligned} \text{Bias}(\hat{F}_{Prop}(t_y)) &\cong \frac{1}{2} F(t_y) \varphi_{02} - \frac{1}{2} F(t_y) \varphi_{11} + \frac{1}{2} m_5 \varphi_{02} + m_6 F(t_y) \\ &\quad + \frac{3}{8} m_6^2 F(t_y) \varphi_{02} - \frac{1}{2} m_6 F(t_y) \varphi_{11}, \\ \text{MSE}(\hat{F}_{Prop}(t_y)) &\cong \frac{1}{4} \varphi_{02} \left\{ (F(t_y) + 2m_5 F(t_x))^2 - 2F(t_y) (3F(t_y) + 4m_5 F(t_x)) m_6 + \right. \\ &\quad \left. 4m_6^2 F^2(t_y) \right\} - F(t_y) \varphi_{11} (1 + m_6) (F(t_y) + 2m_5 F(t_x) + 2m_6 F(t_y)) \\ &\quad + F^2 \{ m_6^2 + \varphi_{20} (1 + m_6)^2 \}. \end{aligned} \quad (25)$$

The optimum values of m_5 , m_6 and m_7 , determined by minimizing (25), are

$$\begin{aligned} m_{5(\text{opt})} &= \frac{F(t_y) \left[-4 \frac{\varphi_{11}}{\varphi_{20}^{-1/2} \varphi_{02}^{1/2}} + \varphi_{02}^{1/2} \left\{ 2 - \varphi_{02} + \varphi_{11} + 2 \left(-1 + \frac{\varphi_{11}^2}{\varphi_{20} \varphi_{02}} \right) \varphi_{20} \right\} \right]}{4F(t_x) \varphi_{02}^{1/2} \left\{ -1 + \left(-1 + \frac{\varphi_{11}^2}{\varphi_{20} \varphi_{02}} \right) \right\}} \quad \text{and} \\ m_{6(\text{opt})} &= \frac{\varphi_{02} - 4 \left(-1 + \frac{\varphi_{11}^2}{\varphi_{20} \varphi_{02}} \right) \varphi_{20}}{4 \left\{ -1 + \left(-1 + \frac{\varphi_{11}^2}{\varphi_{20} \varphi_{02}} \right) \right\}} \end{aligned}$$

respectively. The simplified minimum MSE of $\hat{F}_{Prop}(t_y)$ at the optimum values of m_5 and m_6 is given by

$$\text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) \cong \frac{F^2(t_y) \left\{ 16\varphi_{02}^2 - 8 \left(-1 + \frac{\varphi_{11}^2}{\varphi_{20}\varphi_{02}} \right) (-2 + \varphi_{02}) \varphi_{20} \right\}}{16 \left\{ -1 + \left(-1 + \frac{\varphi_{11}^2}{\varphi_{20}\varphi_{02}} \right) \right\}}. \quad (26)$$

Efficiency comparisons in simple random sampling

In this section, the adapted and proposed estimators of $F(t_y)$ are compared in terms of the minimum MSEs.

1. From (2) and (26),

$$\begin{aligned} \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &< \text{Var}(\hat{F}(t_y)) \text{ if} \\ \text{Var}(\hat{F}(t_y)) - \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &> 0. \end{aligned}$$

2. From (4) and (26),

$$\begin{aligned} \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &< \text{MSE}(\hat{F}_R(t_y)) \text{ if} \\ \text{MSE}(\hat{F}_R(t_y)) - \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &> 0. \end{aligned}$$

3. From (6) and (26),

$$\begin{aligned} \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &< \text{MSE}(\hat{F}_P(t_y)) \text{ if} \\ \text{MSE}(\hat{F}_P(t_y)) - \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &> 0. \end{aligned}$$

4. From (10) and (26),

$$\begin{aligned} \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &< \text{MSE}(\hat{F}_{BT,R}(t_y)) \text{ if} \\ \text{MSE}(\hat{F}_{BT,R}(t_y)) - \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &> 0. \end{aligned}$$

5. From (13) and (26),

$$\begin{aligned} \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &< \text{MSE}(\hat{F}_{BT,P}(t_y)) \text{ if} \\ \text{MSE}(\hat{F}_{BT,P}(t_y)) - \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &> 0. \end{aligned}$$

6. From (17) and (26),

$$\begin{aligned} \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &< \text{Var}_{\min}(\hat{F}_{Reg}(t_y)) \text{ if} \\ \text{Var}_{\min}(\hat{F}_{Reg}(t_y)) - \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &> 0. \end{aligned}$$

7. From (19) and (26),

$$\begin{aligned} \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &< \text{MSE}_{\min}(\hat{F}_{R,D}(t_y)) \text{ if} \\ \text{MSE}_{\min}(\hat{F}_{R,D}(t_y)) - \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &> 0. \end{aligned}$$

8. From (22) and (26),

$$\begin{aligned} \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &< \text{MSE}_{\min}(\hat{F}_{G,K}(t_y)) \text{ if} \\ \text{MSE}_{\min}(\hat{F}_{G,K}(t_y)) - \text{MSE}_{\min}(\hat{F}_{Prop}(t_y)) &> 0. \end{aligned}$$

Empirical study

In this section, we conduct a numerical study to investigate the performances of the adapted and DF estimators. For this purpose, five populations are considered. The summary statistics of these populations are reported in Table 1. The percentage relative efficiency (PRE) of an estimator $\hat{F}_i(t_y)$ with respect to $\hat{F}_{SRS}(t_y)$ is

$$\text{PRE}(\hat{F}_i(t_y), \hat{F}_{SRS}(t_y)) = \frac{\text{Var}(\hat{F}_{SRS}(t_y))}{\text{MSE}_{\min}(\hat{F}_i(t_y))} \times 100,$$

where $i = SRS, P, \dots, Prop$.

The MSEs and PREs of distribution function estimators, computed from five populations, are given in Tables 2 and 3.

Population I [Source: Singh, 2003]

Y : Duration of sleep (in minutes) and X : Age of old persons.

Population II [Source: Gujarati, 2009]

Y : The eggs produced in 1990 (millions) and X : The price per dozen (cents) in 1990

Population III [Source: Murthy, 1967]

Y : The output of the factory and X : The number of workers

Population IV [Source: Sarndal, 1992a]

Y : Population in 1983 (in million) and X : Population in 1980 (in million)

Population V [Source: Koyuncu and Kadilar, 2009]

Y : Number of teachers and X : number of students

Table 1: Summary statistics for Populations I–V

Population	I	II	III	IV	V
N	30	50	80	120	923
n	5	5	10	20	180
λ	0.16667	0.18	0.0875	0.04167	0.00447
$Q_{2(y)}$	387	831	5105	9.25	171
$Q_{2(x)}$	66.5	75.35	148	8.6	4123
$F(t_y)$	0.5	0.5	0.5	0.5	0.50163
C_{Ft_y}	1.01709	1.01015	1.00631	1.00419	0.99729
$F(t_x)$	0.5	0.5	0.5	0.5	0.50054
C_{Ft_x}	1.01709	1.01015	1.00631	1.00419	0.99946
$R_{Ft_y F t_x}$	−0.73333	−0.12000	0.95	0.96667	0.84616
β_2	1	1	1	1	1

Table 2: MSEs using Populations I–V

Estimators	Population-I	Population-II	Population-III	Population-IV	Population-V
$\hat{F}_{SRS}(y)$	0.0431035	0.0459184	0.0221519	0.0105042	0.0011192
$\hat{F}_R(y)$	0.1494253	0.1028571	0.0022152	0.0007003	0.0003451
$\hat{F}_P(y)$	0.0237504	0.0812919	0.0865339	0.0413609	0.0041361
$\hat{F}_{B,R}(y)$	0.0854885	0.0629082	0.0066456	0.0029762	0.0004512
$\hat{F}_{B,P}(y)$	0.0222701	0.0518878	0.0487342	0.0232843	0.0023494
$\hat{F}_{Reg}(y)$	0.0199234	0.0452571	0.0021598	0.0006886	0.0003179
$\hat{F}_{R,D}(y)$	0.0184528	0.0383201	0.0021413	0.0006867	0.0003175
$\hat{F}_{G,K}(y)$	0.0175499	0.0364489	0.0020635	0.0006726	0.0003171
$\hat{F}_{Prop}(y)$	0.0164319	0.0343546	0.0019248	0.0006448	0.0003165

Table 3: PREs using Populations I–V

Estimators	Population-I	Population-II	Population-III	Population-IV	Population-V
$\hat{F}_{SRS}(y)$	100.00	100.00	100.00	100.00	100.00
$\hat{F}_R(y)$	28.85	44.64	1000.00	1500.00	324.29
$\hat{F}_P(y)$	181.49	56.49	25.60	25.40	27.06
$\hat{F}_{B,R}(y)$	50.42	72.99	333.33	352.94	248.08
$\hat{F}_{B,P}(y)$	193.55	88.50	45.45	45.11	47.64
$\hat{F}_{Reg}(y)$	216.35	101.46	1025.64	1525.42	352.09
$\hat{F}_{R,D}(y)$	233.59	119.83	1034.50	1529.63	352.53
$\hat{F}_{G,K}(y)$	245.61	125.98	1073.53	1561.67	353.01
$\hat{F}_{Prop}(y)$	262.32	133.66	1150.86	1629.11	353.67

From the numerical results, presented in Tables 2 and 3, it is noted that the proposed estimator is more precise than the adapted distribution function estimators of f Cochran (1940), Murthy (1964), Bahl and Tuteja (1991), Rao (1991), Singh et al. (2009) and Grover and Kaur (2014), 15in terms of MSE and PRE.

Conclusion

In this paper, we have proposed a new estimator for estimating the finite population distribution function. The biases and MSEs of the proposed estimator were derived using first order approximation. Based on theoretical and numerical comparative studies, it has turned out that the proposed estimator is more precise than their existing counterparts.

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