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A New Generalized Class of Mixture Estimators for Estimating the Population Mean Under Single and Double Phase Sampling Schemes

Sara Azam[1], Hina Khan[2*] and Mustansar Aatizaz[3]
[1,2]Government College, University Lahore, Pakistan
[3]Forman Christan College (A chartered University), Pakistan
[*]Corresponding Author: hinakhan@gcu.edu.pk

Abstract

A new class of mixture estimators is proposed by combining the ratio and regression estimators when the nature of auxiliary variable is qualitative under single and double phase sampling schemes for estimating the population mean. The mean square errors (MSE,s) of the purposed estimators are derived up to the first order of approximation. Finally, we use some real life applications and simulated results to prove that the proposed estimators are more efficient as compared to the several existing estimates.

Keywords: MSE, Auxiliary Variable, Ratio Estimator, Regression Estimator.

Introduction

During any sampling survey, when we are dealing with both the qualitative and quantitative variables, the mixture estimators are found more efficient as compared to the usual estimators for estimating the finite population mean. Several researchers have used the concept of mixture estimators like Singh et al. (2007, 2010), Hammad (2008) , Gupta and Shabbir (2007), Abu-Dayyeh and Dwivedi (2003), Sahoo et al. (1993) and Sisodia and Dwivedi (1981).

Consider a fine population $U= \{U_1, U_2, U_3 \ldots U_N\}$ of size N. A sample of size n is drawn using simple random sampling without replacement (SRSWOR). Let $P= \frac{X}{N}$, is the population proportion in which X represents the number of the occurrence of qualitative variable and $p = \frac{x}{n}$, represents the sample proportion where x represents the number of the occurrence of qualitative variable of sample.

In order to obtain the mean square errors (MSE, s) of the purposed estimators following error terms are used:

In Single phase sampling the terms $\theta_1 = \frac{1-\frac{n_1}{N}}{n_1}$, $E(e_x)=0, E(e_y)=0, E(e_p)=0$, $E(e_x e_y)=\theta\overline{X}\overline{Y}C_x C_y\rho_{xy}, E(e_x e_p)=\theta\overline{X}PC_x C_p\rho_{xp}, E(e_p e_y)=\theta P\overline{Y}C_p C_y\rho_{py}$ are used. In double phase sampling the following terms are used:

$$\theta_2 = \frac{1-\frac{n_2}{N}}{n_2}; E(e_{x_2})=0, E(e_{y_2})=0, E(e_{p_2})=0$$

$$E(e_{x_2}^2)=\theta_2\overline{X}^2C_x^2, E(e_{y_2}^2)=\theta_2\overline{Y}^2C_y^2, E(e_{p_2}^2)=\theta_2P^2C_p^2;$$

$$E(e_{x_1}e_{y_1}) or E(e_{x_2}e_{y_1}) or E(e_{x_1}e_{y_2})=\theta_1\overline{X}\overline{Y}C_xC_y\rho_{xy}; E(e_{x_2}e_{y_2})=\theta_2\overline{X}\overline{Y}C_xC_y\rho_{xy};$$

$$E(e_{x_1}e_{p_1}) or E(e_{x_2}e_{p_1}) or E(e_{x_1}e_{p_2})=\theta_1\overline{X}PC_xC_p\rho_{xp}; E(e_{x_2}e_{p_2})=\theta_2\overline{X}PC_xC_p\rho_{xp};$$

$$E(e_{y_1}e_{p_1}) or E(e_{y_2}e_{p_1}) or E(e_{y_1}e_{p_2})=\theta_1\overline{Y}PC_yC_p\rho_{yp}; E(e_{y_2}e_{p_2})=\theta_2\overline{Y}PC_yC_p\rho_{yp};$$

$$E(e_{x_1}e_{x_2})=\theta_1\overline{X}^2C_x^2; E(e_{y_1}e_{y_2})=\theta_1\overline{Y}^2C_y^2; E(e_{p_1}e_{p_2})=\theta_1P^2C_p^2$$

Mixture Estimators from Literature

In this section several existing estimators of single and double phase sampling schemes are discussed.

Single phase Estimators

Following are the mixture estimators of single phase sampling.

i. Sisodia and Dwivedi (1981) ratio estimator:

$$T_1 = p\frac{\left[\overline{X}+C_x\right]}{\left[\overline{x}+C_x\right]} \tag{1}$$

$$MSE(T_1)=\theta_1P^2\left[C_p^2+C_x^2\alpha(\alpha-2K)\right] \tag{2}$$

where

$$\alpha=\frac{\overline{X}}{\overline{X}+C_x}\quad K=\frac{C_p}{C_x}\rho_{pb}$$

ii. Samiuddin and Hanif(2006) chain ratio type estimator:

$$T_2 = p\left[\frac{\overline{X}}{\overline{\overline{x}}_1}\right]\left[\frac{\overline{Z}}{\overline{\overline{z}}_1}\right] \tag{3}$$

$$MSE(T_2)=\theta_1P^2\left[C_p^2+C_x^2+C_z^2-2C_pC_x\rho_{px}-2C_pC_z\rho_{pz}+2C_xC_z\rho_{xz}\right] \tag{4}$$

iii. Hammad (2008) ratio cum regression estimator:

$$T_3 = \left[p + b_{yx}\left(\overline{X} - \overline{x}_1\right)\right]\left[K\frac{\overline{Z}}{\overline{\overline{z}}_1} + (1-K)\frac{\overline{z}_1}{\overline{\overline{Z}}}\right] \tag{5}$$

$$MSE(T_3) = \theta_1 P^2 C_y^2\left[1 - \rho_{py}^2 - \left(\rho_{pz} - \rho_{pb}\rho_{xz}\right)^2\right] \tag{6}$$

iv. Abu Dayyeh et al. (2003) ratio estimator:

$$T_4 = p\left[\frac{\overline{x}_1}{\overline{\overline{X}}}\right]^{\alpha_1}\left[\frac{\overline{z}_1}{\overline{\overline{Z}}}\right]^{\alpha_2} \tag{7}$$

$$MSE(T_4) = \theta_1 P^2 C_p^2\left[1 - \rho_{y.12}^2\right] \tag{8}$$

Where;

$$\rho_{y.12}^2 = \frac{\rho_{xy}^2 + \rho_{yz}^2 - 2\rho_{py}\rho_{pz}\rho_{xz}}{1 - \rho_{xz}^2}$$

v. Naik and Gupta (1996) ratio estimator:

$$T_5 = \overline{y}_2\frac{P_1}{p_1} \tag{9}$$

$$MSE(T_5) = \theta_1\overline{Y}^2\left[C_y^2 + C_{\tau_1}^2 - 2C_yC_{\tau_1}\rho_{pb_1}\right] \tag{10}$$

vi. Singh et al (2007) ratio type exponential estimator:

$$T_6 = \overline{y}_2\exp\left[\frac{P - p_1}{P + p_1}\right] \tag{11}$$

$$MSE(T_6) = \overline{Y}^2 f_1\left[C_y^2 + C_p^2\left(\frac{1}{4} - K_{pb_2}\right)\right] \tag{12}$$

Where;

$$K_{pb_2} = \rho_{pb_2}\frac{C_y}{C_{p_2}}$$

vii. Shabbir et al (2007) ratio estimator:

$$T_7 = \left[d_1\overline{y} + d_2\left(P - p_1\right)\right]\frac{P}{p_1} \tag{13}$$

$$MSE(T_7) = \frac{\theta\overline{Y}^2 C_y^2\left(1 - \rho_{pb_1}^2\right)}{1 + \theta\left(1 - \rho_{pb_1}^2\right)C_y^2} \tag{14}$$

viii. Singh et al. (2010) ratio estimator:

$$T_8 = \left(\frac{P}{\bar{x}}\right)\bar{X} \tag{15}$$

$$MSE(T_8) = fP^2\left(C_p^2 + C_x^2 - 2C_pC_x\rho_{xp}\right) \tag{16}$$

Double phase Estimators

Following are the mixture estimators of double phase sampling.

i) Srivastava (1971) ratio estimator:

$$T_1^* = p_2\left[\frac{\bar{x}_1}{\bar{x}_2}\right]^{\alpha} \tag{17}$$

$$MSE(T_1^*) = P^2C_p^2\left[\theta_2 - (\theta_2 - \theta_1)\rho_{xp}^2\right] \tag{18}$$

ii) Bedi (1985) ratio estimator:

$$T_2^* = p_2\left[\frac{\bar{z}_1}{\bar{z}_2}\right]^{\alpha} \tag{19}$$

$$MSE(T_2^*) = P^2C_p^2\left[\theta_2 - (\theta_2 - \theta_1)\rho_{zp}^2\right] \tag{20}$$

iii) Chand Sahoo and Sahoo (1993) chain based ratio estimator:

$$T_3^* = p_2\left[\frac{\bar{x}_2}{\bar{x}_1}\right]\left[\frac{\bar{Z}}{\bar{z}_1}\right] \tag{21}$$

$$MSE(T_3^*) = P^2C_p^2\left[\theta_1\left(C_p^2 + C_z^2 - 2C_pC_z\rho_{pz}\right) - (\theta_2 - \theta_1)\left(C_p^2 + C_x^2 - 2C_pC_x\rho_{px}\right)\right] \tag{22}$$

iv) Singh (2001) Chain ratio estimator:

$$T_4^* = p_2\left[\frac{\bar{x}_1}{\bar{x}_2}\right]\left[\frac{a\bar{Z} + \sigma_z}{a\bar{z}_1 + \sigma_z}\right]^{g} \tag{23}$$

$$MSE(T_4^*) = P^2\left[\theta_2C_p^2 + (\theta_2 - \theta_1)\left(C_x^2 - 2C_pC_x\rho_{px}\right) - \theta_1C_p^2\rho_{pz}^2\right] \tag{24}$$

v) Mohanty (1967) ratio cum regression estimator:

$$T_5^* = \left[p_2 + b_{yx}\left(\bar{x}_1 - \bar{x}_2\right)\right]\frac{\bar{Z}}{\bar{z}_2} \tag{25}$$

$$MSE(T_5^*) = P^2\left[\theta_2\left(C_p^2 + C_z^2 - C_p^2\rho_{pb}^2 - 2C_pC_z\rho_{pz} + 2C_pC_z\rho_{pz}\rho_{xz}\right) + \theta_1\left(C_p^2\rho_{pb}^2 - 2C_pC_z\rho_{pz}\rho_{xz}\right)\right] \tag{26}$$

vi) Kiregyera (1980) ratio to regression estimator:

$$T_6^* = \frac{p_2}{\bar{x}_2}\left[\bar{x}_1 + b_{xz}\left(\bar{Z} - \bar{z}_1\right)\right] \tag{27}$$

$$MSE(T_6^*) = P^2\left[\theta_2\left(C_p^2 + C_x^2 - 2C_pC_x\rho_{pb}\right) + \theta_1\left(C_x^2 - C_z^2\rho_{xz}^2 - 2C_pC_x\rho_{pb} + 2C_pC_z\rho_{pb}\rho_{pz}\right)\right] \tag{28}$$

vii) Kharie and Srivasta (1981) ratio cum regression estimator:

$$T_7^* = \left[p_2 + b_{yx}\left(\overline{X} - \overline{x}_1\right)\right]\frac{\overline{Z}}{\overline{z}_2} \tag{29}$$

$$MSE(T_7) = P^2\left[\theta_2\left(C_p^2 + C_z^2 - C_p^2\rho_{pb}^2 - 2C_pC_z\rho_{pz}\right) - \theta_1 C_p\rho_{xy}\left(C_p\rho_{pb} - 2C_z\rho_{xz}\right)\right] \tag{30}$$

viii) Chand (1975) chain ratio type estimator:

$$T_8^* = p_2\left(\frac{\overline{x}_1}{\overline{x}_2}\right)\left(\frac{\overline{Z}}{\overline{z}_1}\right) \tag{31}$$

$$MSE(T_8^*) = P^2\left[\theta_2\left(C_p^2 + C_x^2 - 2C_pC_x\rho_{pb}\right) + \theta_1\left(C_x^2 - C_z^2 - 2C_pC_x\rho_{pb} + 2C_pC_z\rho_{pb}\rho_{pz}\right)\right] \tag{32}$$

ix) Samiuddin and Hanif (2006) chain ratio type estimator:

$$T_9^* = p_2\left(\frac{\overline{x}_1}{\overline{X}}\right)\left(\frac{\overline{z}_1}{\overline{Z}}\right) \tag{33}$$

$$MSE(T_9^*) = P^2\left[\theta_1\left(C_x^2 + 2C_x\left(C_p\rho_{pb} + C_z\rho_{xz}\right)\right) + \theta_2\left(C_p^2 + C_z\left(C_z + 2C_p\rho_{pz}\right)\right)\right] \tag{34}$$

x) Singh et al (2007) ratio type exponential estimator:

$$T_{10}^* = \overline{y}_2 \exp\left[\frac{p_2 - P}{p_2 + P}\right] \tag{35}$$

$$MSE(T_{10}^*) = \overline{Y}^2 f_2\left[C_y^2 + C_p^2\left(\frac{1}{4} + K_{pb_2}\right)\right] \tag{36}$$

Where;

$$K_{pb_2} = \rho_{pb_2}\frac{C_y}{C_{p_2}}$$

Proposed Mixture Estimators for Single Phase

The following single phase mixture estimators have been developed by combining the ratio and regression estimators for no and partial auxiliary information case.

Mixture of Ratio type Exponential Estimator

The following estimator is the mixture ratio estimator in which proportion of qualitative response is used as auxiliary variable and a constant α is also used.

$$t_1 = \bar{y}\exp\left(\frac{P-p}{P+(\alpha-1)p}\right) \tag{37}$$

By expanding the above expression in the term of *'e'* up to the first order of approximation we get the expression:

$$t_1 - \bar{Y} = e_y - \frac{\bar{Y}e_p}{\alpha P} \tag{38}$$

By taking the square of the Equation (38) and applying expectation on both sides we obtain

$$MSE(t_1) = \theta\bar{Y}^2 C_y^2 + \frac{\bar{Y}^2\theta P^2 C_p^2}{\alpha^2 P^2} - 2\frac{\bar{Y}\theta P\bar{Y}C_p C_y\rho_{py}}{\alpha P} \tag{39}$$

Optimizing the equation (39) w.r.t. α , we get:

$$\alpha = \frac{C_p}{C_y\rho_{py}}$$

Optimized MSE of the proposed t_1 w.r.t. α is given as:

$$MSE(t_1)_{\min} = \theta\bar{Y}^2 C_y^2\left(1-\rho_{py}^2\right) \tag{40}$$

Mixture of Ratio cum Regression Type Estimator

The following estimator is the mixture of ratio qualitative estimator and regression quantitative estimator, qualitative estimator is used as auxiliary variable.

$$t_2 = \left[\bar{y} - \alpha\left(\bar{X} - \bar{x}\right)\right]\exp\left[\frac{P-p}{P+(\beta-1)p}\right] \tag{41}$$

Expandingequation(41) in the term of '*e*' up to the first order of approximation we get:

$$t_2 - \bar{Y} = e_y + \alpha e_x - \frac{\bar{Y}e_p}{\beta P} \tag{42}$$

By taking the square of the above Equation (42) and applying expectation on both sides we obtained the following expression:

$$MSE(t_2) = \theta\bar{Y}^2 C_y^2 + \alpha^2\theta\bar{X}^2 C_x^2 + \frac{\bar{Y}^2\theta C_p^2}{\beta^2} + 2\alpha\theta\bar{Y}\bar{X}C_x C_y\rho_{xy} - 2\alpha\frac{\bar{Y}\theta\bar{X}C_x C_p\rho_{xp}}{\beta} - 2\frac{\theta\bar{Y}^2 C_p C_y\rho_{py}}{\beta} \tag{43}$$

By differentiating Equation (43) w.r.t α and β , equate to zero, The optimized values of α and β are given as:

$$\alpha = -\frac{\bar{Y}C_y\left(\rho_{xy} - \rho_{xp}\rho_{py}\right)}{\bar{X}C_x\left(1-\rho_{xp}^2\right)}$$

$$\beta = \frac{C_p\left(1-\rho_{xp}^2\right)}{C_y\left(\rho_{py} - \rho_{xp}\rho_{xy}\right)}$$

By substituting the optimized values α and β in Equation (43) we get the minimum MSE as:

$$MSE(t_2)_{\min} = \theta\bar{Y}^2C_y^2\left[1-\frac{1}{\left(1-\rho_{xp}^2\right)}\left\{\rho_{xy}^2 + \rho_{py}^2 - 2\rho_{xy}\rho_{xp}\rho_{py}\right\}\right] \quad (44)$$

Mixture Estimators for Double Phase

Following are the mixture estimators for the double phase.

Mixture of Ratio Estimator for No Information

The following estimator is the mixture ratio estimator for no information case,Four auxiliary variables are used two of them are qualitative and other two are quantitative.

$$t_3 = \bar{y}_2 + \alpha_1\bar{x}_1 + \alpha_2\bar{x}_2 + \beta_1 p_1 + \beta_2 p_2 \quad (45)$$

By expanding the above Equation (45) in the term of *'e'* up to the first order of approximation and suppose that $\alpha_1 + \alpha_2 = 0$ and $\beta_1 + \beta_2 = 0$ so, by substituting $\alpha_2 = -\alpha_1$ and $\beta_2 = -\beta_1$ we get:

$$t_3 - \bar{Y} = e_{y_2} + \alpha_1\left(e_{x_1} - e_{x_2}\right) + \beta_1\left(e_{p_1} - e_{p_2}\right) (46)$$

By taking the square of the Equation (46) and applying expectation on both sides we obtain:

$$MSE(t_3) = \begin{bmatrix} \theta_2\bar{Y}^2C_y^2 + \alpha_1^2\bar{X}^2C_x^2\left(\theta_2-\theta_1\right) + \beta_1^2P^2C_p^2\left(\theta_2-\theta_1\right) + 2\alpha_1\bar{X}\bar{Y}C_xC_y\rho_{xy}\left(\theta_1-\theta_2\right) + \\ 2\beta_1P\bar{Y}C_pC_y\rho_{py}\left(\theta_1-\theta_2\right) + 2\alpha_1\beta_1\bar{X}PC_xC_p\rho_{xp}\left(\theta_2-\theta_1\right) \end{bmatrix} (47)$$

By differentiating above Equation (47) w.r.t α_1 and β_1 and equate to zero. The optimized value of α_1 and β_1 is given as:

$$\alpha_1 = \frac{\bar{Y}C_y\left(\rho_{xy} - \rho_{yp}\rho_{xp}\right)}{\bar{X}C_x\left(1-\rho_{xp}^2\right)}$$

$$\beta_1 = \frac{\bar{Y}C_y\left(\rho_{py} - \rho_{xy}\rho_{xp}\right)}{PC_p\left(1-\rho_{xp}^2\right)}$$

By Substituating the optimized values of α_1 and β_1 in the Equation (47) the minimum expression of MSE is given as:

$$MSE(t_3)_{\min} = \bar{Y}^2C_y^2\left[\theta_2 - \frac{\left(\theta_2-\theta_1\right)}{\left(1-\rho_{xp}^2\right)}\left\{\rho_{xy}^2 - 2\rho_{xy}\rho_{xp}\rho_{yp} + \rho_{py}^2\right\}\right] \quad (48)$$

Mixture of Ratio Estimator for Partial information

The following estimator is the mixed ratio estimator for estimating the population mean. In this estimator simple ratio estimator is used by taking two auxiliary variables, one is quantitative (x) and the other is qualitative (p). This estimator is the form of partial information.

$$t_4 = \overline{y_2} \frac{p_2}{p_1} \frac{\overline{X}}{\overline{x_1}} \tag{49}$$

By expanding the above expression in the term of *'e'* up to the first order of approximation we get the expression:

$$t_4 - \overline{Y} = e_{y_2} - \frac{\overline{Y} e_{x_1}}{\overline{X}} + \frac{\overline{Y}\left(e_{p_2} - e_{p_1}\right)}{P} \tag{50}$$

By taking the square of the above the Equation (50) and applying expectation on both sides we obtained:

$$MSE(t_4) = \overline{Y}^2 \left[\theta_2 C_y^2 + \theta_1 \left\{ \left(C_x - C_y \rho_{xy}\right)^2 - \left(C_y \rho_{xy}\right)^2 \right\} + \left(\theta_2 - \theta_1\right) \left\{ \left(C_p + C_y \rho_{py}\right)^2 - \left(C_y \rho_{py}\right)^2 \right\} \right] \tag{51}$$

Mixture of Ratio type Exponential Estimator for Partial Information

The following estimator is the mixture of exponential ratio type estimator for partial information. In which proportion of qualitative response is used as auxiliary variable.

$$t_5 = \overline{y_2} \exp\left(\frac{P - p_1}{P + \left(\alpha - 1\right) p_1} \right) \tag{52}$$

By expanding the above expression in the term of *'e'* up to the first order of approximation we get the expression:

$$t_5 - \overline{Y} = e_{y_2} - \frac{\overline{Y} e_{p_1}}{\alpha P} \tag{53}$$

By taking the square of the above the Equation (53) and applying expectation on both sides we obtain

$$MSE\left(t_5\right) = \theta_2 \overline{Y}^2 C_y^2 + \frac{\overline{Y}^2 \theta_1 C_p^2}{\alpha^2} - 2 \frac{\theta_1 \overline{Y}^2 C_p C_y \rho_{py}}{\alpha} \tag{54}$$

Differentiating above Equation (54) w.r.t α. The optimize value of α is given as:

$$\alpha = \frac{C_p}{C_y \rho_{py}}$$

Substituting the optimized value of α we get the minimum MSE as:

$$MSE\left(t_5\right)_{\min} = \overline{Y}^2 C_y^2 \left(\theta_2 - \theta_1 \rho_{py}^2\right) \tag{55}$$

Mixture of Chain Ratio Estimator for Partial Information

In this section the mixed chain ratio type estimator is developed using three auxiliary variables under partial information.

$$t_6 = \overline{y}_2 \left[\frac{\overline{x}_1}{\overline{\overline{x}}_2}\right]^{\alpha_1} \left[\frac{\overline{X}}{\overline{\overline{x}}_2}\right]^{\alpha} \left[\frac{p_1}{p_2}\right]^{\beta} \tag{56}$$

By expanding the above expression in the term of *'e'* up to the first order of approximation and suppose that $\alpha_1 + \alpha = 0$ so $\alpha = -\alpha_1$ we get the expression:

$$t_6 - \overline{Y} = e_{y_2} + \frac{\beta \overline{Y}\left(e_{p_1} - e_{p_2}\right)}{P} + \frac{\alpha_1 \overline{Y} e_{x_1}}{\overline{X}} \tag{57}$$

By taking the square of the above Equation (57) and applying expectation on both sides we obtained:

$$MSE(t_6) = \theta_2 \overline{Y}^2 C_y^2 + \beta^2 \overline{Y}^2 C_p^2\left(\theta_2 - \theta_1\right) + \alpha^2 \theta_1 \overline{Y}^2 C_x^2 + 2\beta \overline{Y}^2 C_y C_p \rho_{yp}\left(\theta_1 - \theta_2\right) + 2\theta_1 \alpha_1 \overline{Y}^2 C_y C_x \rho_{xy} \tag{58}$$

By differentiating the Equation w.r.t α_1 and β. The optimized values of α_1 and β are given as:

$$\alpha_1 = \frac{-C_y \rho_{xy}}{C_x}, \qquad \beta = \frac{C_y \rho_{yp}}{C_p}$$

By putting the optimized values of α_1 and β we get the minimum MSE as:

$$MSE(t_6)_{\min} = \overline{Y}^2 C_y^2\left[\theta_2 - \left(\theta_2 - \theta_1\right)\rho_{yp}^2 - \theta_1 \rho_{xy}^2\right] \tag{59}$$

Mixture of Ratio cum Regression for No information:

The suggested estimator is the mixture of qualitative and quantitative auxiliary variable of ratio cum regression type estimator for no information.

$$t_7 = \left[\overline{y}_2 - \alpha\left(\overline{x}_1 - \overline{x}_2\right)\right] \exp\left[\frac{p_1 - p_2}{p_1 + (\beta - 1) p_2}\right] \tag{60}$$

By expanding the above expression in the term of *'e'* up to the first order of approximation we get:

$$t_7 - \overline{Y} = e_y - \alpha\left(e_{x_1} - e_{x_2}\right) + \frac{\overline{Y}\left(e_{p_1} - e_{p_2}\right)}{P\beta} \tag{61}$$

By taking the square of the Equation (61) and applying expectation on both sides we obtained:

$$MSE(t_7) = \left[\begin{array}{l} \theta_2 \overline{Y}^2 C_y^2 + \alpha^2 \overline{X}^2 C_x^2\left(\theta_2 - \theta_1\right) + \dfrac{\overline{Y}^2 C_p^2\left(\theta_2 - \theta_1\right)}{\beta^2} + 2\alpha \overline{X}\,\overline{Y} C_x C_y \rho_{xy}\left(\theta_2 - \theta_1\right) - \\ 2\dfrac{\overline{Y}^2 C_p C_y \rho_{py}\left(\theta_2 - \theta_1\right)}{\beta} - 2\dfrac{\alpha \overline{Y}\,\overline{X} C_p C_x \rho_{px}\left(\theta_2 - \theta_1\right)}{\beta} \end{array}\right] \tag{62}$$

By differentiating the Equation w.r.t α and β . The optimized values of α and β are given as:

$$\alpha = -\frac{\bar{Y} C_y \left(\rho_{xy} - \rho_{xp}\rho_{py}\right)}{\bar{X} C_x \left(1 - \rho_{xp}^2\right)}$$

$$\beta = \frac{C_p \left(1 - \rho_{xp}^2\right)}{C_y \left(\rho_{py} - \rho_{xp}\rho_{xy}\right)}$$

By substituting the optimized values α and β of we get the minimum MSE as:

$$MSE(t_7)_{\min} = \bar{Y}^2 C_y^2 \left[\theta_2 - \frac{(\theta_2 - \theta_1)}{\left(1 - \rho_{xp}^2\right)} \left\{\rho_{py}^2 + \rho_{xy}^2 - 2\rho_{xy}\rho_{xp}\rho_{py}\right\}\right] \tag{63}$$

4. Numerical Analysis:

In this section three real life datasets are used to assess the performance of the proposed estimator as compared to several existing estimators. Estimators are compared on the basis of mean square error's (MSE's). The description of data set of the population is given below.

Table 1: Summary Statistics

Parameters	**Population I**				**Population II**				**Population III**			
	Source: Kutner et.all (2004), page 1350				Source: Kutner et.all (2004), page 1352				Source: Kutner et.all (2004), page 1353			
	X	**Y**	**Z**	**P**	**X**	**Y**	**Z**	**P**	**X**	**Y**	**Z**	**P**
	Market Share	Price	Gross Neilison Rating Point	Discount Price	Team Exp.	Website del.	Backlog of order	Process Change	Sale Price	Finish Square feet	Lot size	Air cond.
	N=36, n_1 =25, n_2 =15				N=73, n_1=50, n_2 =25				N=522, n_1=350, n_2 =150			
$\bar{X}$	2.67				10.84				277894.14			
$\bar{Y}$	2.323				9.04				2262.60			
$\bar{Z}$	288.05				27.82				24369.7			
P	0.58				0.36				0.83			
C_x	0.099				0.522				0.492			
C_y	0.070				0.7835				0.314			
C_z	0.4341				0.2867				0.47			
C_p	288.8				22.39				14053.2			
ρ_{xy}	-0.1832				0.44				0.817			
ρ_{xp}	0.0023				0.039				0.00001			
ρ_{yp}	-0.000045				0.042				0.00001			
ρ_{xz}	0.0705				0.7529				0.2237			
ρ_{yz}	-0.377				0.36				0.1572			
ρ_{zp}	-0.00021				0.039				-0.00001			

The comparison of the existing mixture estimator and purposed mixture estimators is done on the basis of relative efficiency. Relative efficiency can be computed by:

$$\text{Relative Efficiency (R.E)} = \frac{Existing\ Estimator}{Purposed\ Estimator} * 100$$

Single Phase sampling scheme

Two mixture ratio and regression type estimators are proposed under single phase sampling scheme and compared them with several existing estimators.

Table 2: Mixture of Ratio type Exponential Estimator

Existing Estimators	Population I	Population II	Population III
	$R.E(t_1)$	$R.E(t_1)$	$R.E(t_1)$
Abu-Dayyeh (2003)	88179012.3	127.002	27000.377
Singh et al. (2007)	416.7	13787.87	27000.377
Naik et al. (1996)	107094135.8	20441.4	20441.4
Gupta (1996)	425098765.4	82177.6	1.99613E+11
Singh et al. (2010)	107094228.4	126.79	27000.377
Sisodia (1981)	107094135.8	127.002	27000.377

Table 3: Mixture Ratio cum Regression Estimator

Existing Estimators	Population I	Population II	Population III
	$R.E(t_2)$	$R.E(t_2)$	$R.E(t_2)$
Shabbir and Gupta (2007)	103.5	123.3909	302.0617
Hammad (2008)	110608862	156.9129	81565.23

In the Tables 2 and 3, we obtain the PRE'sto assess the performance of all considered estimators under single phase sampling scheme. In the light of the above results, the proposed estimators $\boldsymbol{t_1}$ and $\boldsymbol{t_2}$are more efficient as compared to the usual estimators

Double Phase sampling Scheme

In this section the proposed mixture estimators under two phase sampling scheme are compared with several existing estimators.

Table 4: Mixture Ratio Estimator for no information

Existing Estimators	Population I	Population II	Population III
	$R.E(t_3)$	$R.E(t_3)$	$R.E(t_3)$
Singh et al. (2010)	3835367.88	1515.26	238.07
Naik et al.(1996)	1205377.61	2246.36	247.12

Table 5: Mixture Ratio Estimator for partial information

Existing Estimators	Population I	Population II	Population III
	$R.E(t_4)$	$R.E(t_4)$	$R.E(t_4)$
Singh et al. (2010)	164.05	123.39	1815.04
Naik et al.(1996)	114.49	152.67	9159.63

Table 6: Mixture Ratio type Exponential Estimator

Existing Estimators	Population I	Population II	Population III
	$R.E(t_5)$	$R.E(t_5)$	$R.E(t_5)$
Abu-Dayyeh (2003)	27657308.81	30.3894	5350.304
Singh et al. (2007)	130.687	4891.092	5350.304
Naik et al. (1996)	33590029.04	19549.36	39554658629
Gupta (1996)	33590029.04	6229351	39554658629
Singh et al. (2010)	33590058.08	30.337	3810536.5
Sisodia (1981)	33590029.04	30.3894	5350.304

Table 7: Comparison of Mixture Chain Ratio Estimator

Existing Estimators	Population I	Population II	Population III
	$R.E(t_6)$	$R.E(t_6)$	$R.E(t_6)$
Sahoo (1993)	1104.0466	1.676345	648589.467
Singh (2001)	1104.0432	1.671723	648589.466

Table 8: Comparison of Mixture Ratio cum Regression Estimator

Existing Estimators	Population I	Population II	Population III
	$R.E(t_7)$	$R.E(t_7)$	$R.E(t_7)$
Mohanty (1967)	104549242.4	110.3745	17573.76
Kiregyera (1980)	104713068.2	125.378	17748.08
Kharie (1967)	104562812.5	110.347	17573.78
Samiuddin (2006)	104551136.4	110.3977	17573.76

In Tables 4-8, we obtain the PRE values to assess the performances of all considered estimators under double phase sampling scheme. In light of the above results we see that, the proposed estimators $\boldsymbol{t_3}$ to$\boldsymbol{t_7}$are more efficient as compared to the existing estimators.

Simulation Study

In this section, simulated study is conducted using two real life population to evaluate the performances of estimators. Populations of size 36 and 73 for both the populations, respectively under single and double sampling schemes. We simulate the results 50,000 times to reach out the results. The comparison of the existing mixture estimator and purposed mixture estimators is done on the basis of relative efficiency.

$$\text{Relative Efficiency (R.E)} = \frac{\text{Existing Estimator}}{\text{Purposed Estimator}} * 100$$

Single Phase sampling scheme

We have proposed two mixture estimators under single phase sampling scheme and compared them with several existing estimators.

Table 9: Mixture of Ratio type Exponential Estimator

Estimators	Population I	Population II
t_1	100	100
Chand(1975)	757198582	1.74563E+06
Singh et el (2007)	1349975	1615.074
Abu_Dayyeh et al(2003)	3.181581E+24	1705330.807
Sisodia et al(1981)	7.186851E+08	1681309.983
Singh et el (2010)	682019878	1684516.82

Table 10: Comparison of Mixture Ratio cum Regression Estimator

Estimators	Population I	Population II
t_2	100	100
Mohanty (1967)	241071.4	3.921E+08
Hammad(2008)	279363.4	385019605

In Tables 9 and 10, we obtain the PRE values to through simulation study and assess the performances of all considered estimators under single phase sampling scheme. And observed that, the proposed estimators $\boldsymbol{t_1}$ and$\boldsymbol{t_2}$are more efficient as compared to the usual estimators.

Double Phase sampling scheme

We have proposed five mixture estimators under double phase sampling scheme and compared them with several existing estimators.

Table 11: Mixture Ratio Estimator for No Information

Estimators	Population I	Population II
t_3	100	100
Singh et el (2010)	2012688.36	7.279E+02
Sammudin(2006)	2092532.48	732.2178398
Sahoo et al (1993)	2100143.41	750.665114

Table 12: Mixture Ratio Estimator for and partial information

Estimators	Population I	Population II
t_4	100	100
Singh et al (2010)	10961.1332	2.345E+03

Table 13: Mixture Ratio type Exponential Estimator

Estimators	Population I	Population II
t_5	100	100
Abu_Dayyeh et al(2003)	337198.2681	2.938E+04
Singh et el (2010)	188115.3012	29443.90362
Sisodia et al(1981)	198150.1393	29375.01857

Table 14: Comparison of Mixture Chain Ratio Estimator

Estimators	Population I	Population II
t_6	100	100
Sahoo et al (1993)	827311.3	8.752E+03
Sammudin(2006)	794319	8689.875

Table 15: Mixture Ratio cum Regression Estimator

Estimators	Population I	Population II
t_7	100	100
Mohanty et al (1967)	4410959	9.284E+02
Sammudin et al (2006)	4463695	919.8719
Kiregyera(1980)	4549928	939.5643
Khaire et al(1981)	4402302	940.0201

In Tables 11-15, we obtain the PRE values to through simulation study and assess the performances of all considered estimators under single phase sampling scheme. The results showsthat, the proposed estimators t_3 to $t_7$are more efficient as compared to the usual estimators.

Conclusion

A new class of estimators is proposed by combining the ratio and regression estimators when the nature of auxiliary variable is qualitative under phase one and phase two sampling schemes for estimating the population mean using the auxiliary information. The purposed estimators are compared with Naik et al. (1996), Shabbir and Gupta (2007), Hammad (2008), Gupta and Shabbir (1996), Singh et al. (2010), Sisodia (1981), Abu-Dayyeh (2003), Sahoo (1993) and Singh et al. (2007) mixture estimators. The suggested mixture estimator provides the minimum mean square error and the relative efficiency of each existing estimator is greater than 100 for each population, which shows the superiority of the proposed estimators in all situations. By observing all the above results one may conclude that the purposed mixture estimators provide the better results for single phase and also for double phase sampling scheme.

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