A Class of Estimator for Population Mean Under SRS

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Abstract

In this paper we propose a class of estimators for the estimation of finite population mean using the auxiliary information when SRS scheme is used. The expressions for the Bias and mean square error (MSE) of the existing and suggested class of estimators are derived up to first degree of approximation and the efficiency comparison of suggested class of estimators is made with other existing estimators, using both theoretically and numerically based on real population sets.

Key words: Auxiliary variable, Bias, Mean square error, relative efficiency

Notations and Symbols

Let we have a population of fixed size N such that $U = \{U_1, U_2, \dots, U_N\}$. Each $U_i = (Y_i, X_i)$, $i = 1, 2, 3, \dots, N$, has a pair of values. Here Y is the variable under study which is correlated with the auxiliary variable X . let $y = \{y_1, y_2, \dots, y_n\}$ and $x = \{x_1, x_2, \dots, x_n\}$ be n sample selected by using simple random sampling without replacement (SRSWOR), then consider the following notations

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_i \quad \text{Population mean of the studyvariable Y}$$

$$\bar{X} = \frac{1}{N} \sum_{i=1}^N X_i \quad \text{Population meanof theauxiliaryvariable X}$$

$$S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (Y_i - \bar{Y})^2 \quad \text{Populationvariance of the StudyvariableY}$$

$$S_x^2 = \frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})^2 \quad \text{Population variance of the auxiliary variable X}$$

$$S_{yx} = \frac{1}{N-1} \sum_{i=1}^N (Y_i - \bar{Y})(X_i - \bar{X}) \quad \text{Population covariance among Y and X}$$

$$C_y = \frac{S_y}{\bar{Y}} \quad \text{Population CV of the study variable Y}$$

$$C_x = \frac{S_x}{\bar{X}} \quad \text{Population CV of the auxiliary variable X}$$

$$\rho = \frac{C_{yx}}{C_y C_x} \quad \text{Population Correlation coefficient between Y and X}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \quad \text{Coefficient of skewness of the auxiliary variable X}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \quad \text{Coefficient of kurtosis of the auxiliary variable X}$$

$$\text{Md} \quad \text{Median of the auxiliary variable X}$$

$$\lambda = \left(\frac{1}{n} - \frac{1}{N} \right) \quad \text{FPCF}$$

The following useful error terms are considered to derive the expressions for the Bias and MSE of the estimators

$$e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}} \quad e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}} \quad \text{Such that} \quad \bar{y} = \bar{Y}(1 + e_0) \quad \text{and} \quad \bar{x} = \bar{X}(1 + e_1)$$

The expected values of the above error terms are given as

$$E(e_0) = 0 \quad E(e_1) = 0$$

$$E(e_0^2) = \lambda C_y^2 \quad E(e_1^2) = \lambda C_x^2 \quad E(e_0 e_1) = \lambda \rho C_y C_x$$

Where ρ is the coefficient of correlation between study variable Y and the auxiliary variable X.

Existing Estimators

Here we will discourse a number of existing estimators of the population mean available in literature. The expressions for the bias and MSE of each of these estimators up to the first order of approximation are also given

The usual mean estimator of the study variable is

$$t_n = \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i \quad (1)$$

The variance of t_n is given by

$$Var(t_n) = \lambda \bar{Y}^2 C_y^2 \quad (2)$$

The ratio estimator is given by

$$t_r = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right) \quad (3)$$

The bias and MSE of t_r are given by

$$Bias(t_r) = \lambda \bar{Y}^2 (C_y^2 - \rho C_y C_x)$$

$$\text{and } MSE(t_r) = \lambda \bar{Y}^2 (C_y^2 + C_x^2 - 2\rho C_y C_x) \quad (4)$$

In case of negatively correlation, the product estimator is given by

$$t_p = \bar{y} \left(\frac{\bar{x}}{\bar{X}} \right) \quad (5)$$

Bias and MSE of t_p are given by

$$Bias(t_p) = \lambda \bar{Y} \rho C_y C_x$$

$$\text{and } MSE(t_p) = \lambda \bar{Y}^2 (C_y^2 + C_x^2 + 2\rho C_y C_x) \quad (6)$$

Many researchers modified the above estimators by using the further information of auxiliary variable like median, quartiles, coefficient of skewness and kurtosis etc. Al-Omari et al. (2009) proposed the following ratio type estimator, by using the known first quartile of the auxiliary variable

$$t_{q1} = \bar{y} \left(\frac{\bar{X} + Q_1}{\bar{x} + Q_1} \right) \quad (7)$$

The bias and MSE of t_{q1} are given by

$$Bias(t_{q1}) = \lambda \bar{Y} (C_x^2 \theta_1^2 - \rho \theta_1 C_y C_x)$$

$$\text{and } MSE(t_{q1}) = \lambda \bar{Y}^2 (C_y^2 + C_x^2 \theta_1^2 - \rho \theta_1 C_y C_x) \quad (8)$$

where

$$\theta_1 = \frac{\bar{X}}{\bar{X} + Q_1}$$

Bahl and Tuteja (1991) proposed the following ratio type estimator

$$t_{\text{exp}} = \bar{y} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \quad (9)$$

Bias and MSE of t_{exp} are given by

$$Bias(t_{\text{exp}}) = \lambda \bar{Y} \left(\frac{3}{8} C_y^2 - \frac{1}{2} \rho C_y C_x \right)$$

$$\text{and } MSE(t_{\text{exp}}) = \lambda \bar{Y}^2 \left(C_y^2 + \frac{1}{4} C_x^2 - \rho C_y C_x \right) \quad (10)$$

Bahl and Tuteja (1991) proposed the following product type estimator

$$t_{\text{exp2}} = \bar{y} \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right) \quad (11)$$

Bias and MSE of t_{exp2} are given by

$$Bias(t_{\text{exp2}}) = \lambda \bar{Y} \left(-\frac{1}{8} C_x^2 + \frac{1}{2} \rho C_y C_x \right)$$

$$\text{and } MSE(t_{\text{exp2}}) = \lambda \bar{Y}^2 \left(C_y^2 + \frac{1}{4} C_x^2 + \rho C_y C_x \right) \quad (12)$$

By combining ratio and product estimator, Singh and Espejo (2003) proposed the following estimator

$$t_{\text{rp}} = \bar{y} \left[\alpha_1 \left(\frac{\bar{X}}{\bar{x}} \right) + (1 - \alpha_1) \left(\frac{\bar{x}}{\bar{X}} \right) \right] \quad (13)$$

Where α_1 is a constant with the optimum value as given by

$$\alpha_{1_{\text{opt}}} = \frac{1}{2} + \frac{1}{2} \frac{\rho C_y}{C_x}$$

Bias and MSE of t_{rp} are given as

$$Bias(t_{rp}) = \lambda \bar{Y} \left(\left(\frac{1}{2} + \frac{1}{2} \rho \frac{C_y}{C_x} \right) C_x^2 - \rho^2 C_y^2 \right)$$

$$\text{and } MSE(t_{rp}) = \lambda \bar{Y}^2 C_y^2 (1 - \rho^2) \quad (14)$$

Which is equivalent to the min MSE of the linear regression estimator

Tailor and Sharma (2009) reformed the Singh and Espejo (2003) estimator by using the known C.V and coefficient of kurtosis of the auxiliary variable as

$$t_{ts} = \bar{y} \left[\alpha_2 \left(\frac{C_x \bar{X} + \beta_2}{C_x \bar{x} + \beta_2} \right) + (1 - \alpha_2) \left(\frac{C_x \bar{x} + \beta_2}{C_x \bar{X} + \beta_2} \right) \right] \quad (15)$$

where α_2 is constant with the following optimum value

$$\alpha_{2_{opt}} = \frac{1}{2} \frac{\rho C_y}{\theta_2 C_x}$$

Bias and MSE of t_{ts} are given by

$$Bias(t_{ts}) = -\lambda \bar{Y} \rho C_y \left(\rho C_y - \frac{3}{2} \theta_2 C_x^2 \right)$$

$$MSE(t_{ts}) = \lambda \bar{Y}^2 C_y^2 \left[(1 - \rho^2) C_y^2 - C_x^2 \theta_2^2 \right] \quad (16)$$

where

$$\theta_2 = \frac{\bar{X} C_x}{\bar{X} C_x + \beta_2}$$

The min MSE of t_{ts} for the optimum value of α_2 is equivalent to the min MSE of linear regression estimator i.e.

$$Min MSE(t_{ts}) = \lambda \bar{Y}^2 C_y^2 (1 - \rho^2)$$

Kumar (2015) used the known median of auxiliary variable and reformed Singh and Espejo (2003) estimator as

$$t_k = \bar{y} \left[\alpha_3 \left(\frac{\bar{X} + Md}{\bar{x} + Md} \right) + (1 - \alpha_3) \left(\frac{\bar{x} + Md}{\bar{X} + Md} \right) \right] \quad (17)$$

where α_3 is constant with the following optimum value

$$\alpha_{3_{opt}} = \frac{1}{2} \frac{\rho C_y}{\theta_3 C_x}$$

Bias and MSE of t_k , are as below

$$\begin{aligned} Bias(t_k) &= -\lambda \bar{Y} \rho C_y \left(\rho C_y - \frac{3}{2} \theta_3 C_x^2 \right) \\ MSE(t_k) &= \lambda \bar{Y}^2 C_y^2 \left[(1 - \rho^2) C_y^2 - C_x^2 \theta_3^2 \right] \end{aligned} \quad (18)$$

where

$$\theta_3 = \frac{\bar{X}}{\bar{X} + Md}$$

The min MSE of t_k for the optimum value of α_3 is equivalent to the min MSE of linear regression estimator i.e

$$Min MSE(t_k) = \lambda \bar{Y}^2 C_y^2 (1 - \rho^2)$$

The very known linear regression estimator is

$$t_{reg} = \bar{y} + b_{yx} (\bar{X} - \bar{x}) \quad (19)$$

Where $b_{yx} = \frac{s_{yx}}{s_x^2}$ is coefficient of sample regression. The bias and MSE of t_{reg} are given by

$$Bias(t_{reg}) = -\lambda \beta S_x \left(\frac{\lambda_{12}}{\rho} - \lambda_{03} \right)$$

$$\min MSE(t_{reg}) = \lambda \bar{Y}^2 C_y^2 (1 - \rho^2) \quad (20)$$

Singh et al. (2008) suggested a ratio-cum-product exponential type estimator as

$$t_s = \bar{y} \left[\alpha_4 \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) + (1 - \alpha_4) \exp \left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right) \right] \quad (21)$$

Where α_4 constant whose optimum value is given by

$$\alpha_{4_{opt}} = \frac{1}{2} + \frac{\rho C_y}{C_x}$$

The bias and MSE of t_s are given by

$$Bias(t_s) = -\frac{1}{8} \lambda \bar{Y} (8\rho^2 C_y^2 - 4\rho C_y C_x - C_x^2)$$

$$Min MSE(t_s) = \lambda \bar{Y}^2 C_y^2 (1 - \rho^2) \quad (22)$$

Which is equivalent to the min MSE of linear Regression estimator

Proposed Class of Estimators

Our proposed class of estimators is in the line of Searls (1964). Here we propose the following class of estimators

$$t_{p1} = k_1 \bar{y} \left[\alpha_5 \left(\frac{\bar{X}}{\bar{x}} \right) + (1 - \alpha_5) \left(\frac{\bar{x}}{\bar{X}} \right) \right] \quad (23)$$

$$t_{p2} = k_2 \bar{y} \left[\alpha_6 \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) + (1 - \alpha_6) \exp \left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right) \right] \quad (24)$$

where k_1, k_2, α_5 and α_6 are constants

Now solving equation (23)

$$\begin{aligned} t_{p1} &= k_1 \bar{Y} (1 + e_0) \left[\alpha_5 (1 + e_1)^{-1} + (1 - \alpha_5) (1 + e_1) \right] \\ t_{p1} &= k_1 \bar{Y} \left[1 + e_0 + (1 - 2\alpha_5)(e_1 + e_0 e_1) + \alpha_5 e_1^2 \right] \\ t_{p1} - \bar{Y} &= \bar{Y} \left[(k_1 - 1) + k_1 \{ e_0 + (1 - 2\alpha_5)(e_1 + e_0 e_1) + \alpha_5 e_1^2 \} \right] \end{aligned} \quad (25)$$

Bias and MSE of t_{p1} are given by

$$Bias(t_{p1}) = \bar{Y} E \left[(k_1 - 1) + \lambda k_1 \{ (1 - 2\alpha_5) \rho C_y C_x + \alpha_5 C_x^2 \} \right]$$

and

$$\begin{aligned} MSE(t_{p1}) &= \bar{Y}^2 (k_1 - 1)^2 + \lambda \bar{Y}^2 k_1^2 \left\{ C_y^2 + (1 - 2\alpha_5)^2 C_x^2 + 2(1 - 2\alpha_5) \rho C_y C_x \right\} + 2\lambda \bar{Y}^2 k_1 (k_1 - 1) \\ &\quad \left\{ \alpha_5 C_x^2 + (1 - 2\alpha_5) \rho C_y C_x \right\} \end{aligned} \quad (26)$$

Now minimizing (26) w.r.t α_5 and k_1 , up to zero, we get the following two solution set

Solution set I

$$k_1 = 0 \text{ and } \alpha_5 = \frac{\lambda \rho C_y C_x + 1}{\lambda C_x (2 \rho C_y - C_x)}$$

Solution set II

$$k_1 = \frac{8 \lambda \rho^2 C_y^2 - 6 \lambda \rho C_y C_x - \lambda C_x^2 - 4}{16 \lambda \rho^2 C_y^2 - 8 \lambda \rho C_y C_x - 3 \lambda C_x^2 - 4 \lambda C_y^2 - 4}$$

$$\text{and } \alpha_5 = \frac{4 \lambda \rho^2 C_x C_y^2 - 3 \lambda \rho C_y C_x^2 + 2 \lambda \rho C_y^3 - \lambda C_x^3 - \lambda C_x C_y^2 - 2 \rho C_y - 2 C_x}{C_x (8 \lambda \rho^2 C_y^2 - 6 \lambda \rho C_y C_x - \lambda C_x^2 - 4)}$$

by substituting the values of solution set II, we get minimum bias and MSE as

$$Bias(t_{p1}) = \frac{\lambda \bar{Y} (4 \lambda \rho^2 C_x^2 C_y^2 - 4 \lambda \rho^2 C_y^4 - 2 \lambda \rho C_y C_x^3 + 4 \lambda \rho C_x^3 C_y - \lambda C_x^4 - \lambda C_x^2 C_y^2 - 4 \rho^2 C_y^2 + 4 C_y^2)}{4 + 4 \lambda C_y^2 + 3 \lambda C_x^2 + 8 \lambda \rho C_x C_y - 16 \lambda \rho^2 C_y^2}$$

and

$$MSE(t_{p1}) = \frac{\lambda \bar{Y}^2 (4 \lambda \rho^2 C_x^3 C_y^2 - 4 \lambda \rho^2 C_y^4 - 2 \lambda \rho C_y C_x^3 + 4 \lambda \rho C_x^3 C_y - \lambda C_x^4 - \lambda C_x^2 C_y^2 - 4 \rho^2 C_y^2 + 4 C_y^2)}{4 + 4 \lambda C_y^2 + 3 \lambda C_x^2 + 8 \lambda \rho C_x C_y - 16 \lambda \rho^2 C_y^2} \quad (27)$$

Now solving (24)

$$t_{p2} = k_2 \bar{y} \left[\alpha_6 \exp \left\{ \frac{-e_1}{2} \left(1 + \frac{e_1}{2} \right)^{-1} \right\} + (1 - \alpha_6) \exp \left\{ \frac{e_1}{2} \left(1 + \frac{e_1}{2} \right)^{-1} \right\} \right]$$

$$t_{p2} = k_2 \bar{y} \left[\alpha_6 \exp \left\{ -\frac{e_1}{2} + \frac{e_1^2}{4} \right\} + (1 - \alpha_6) \exp \left\{ \frac{e_1}{2} - \frac{e_1^2}{4} \right\} \right]$$

$$t_{p2} = k_2 \bar{y} \left[\alpha_6 \left\{ 1 - \frac{e_1}{2} + \frac{e_1^2}{4} + \frac{e_1^2}{8} \right\} + (1 - \alpha_6) \left\{ 1 + \frac{e_1}{2} - \frac{e_1^2}{4} + \frac{e_1^2}{8} \right\} \right]$$

$$t_{p2} - \bar{Y} = \bar{Y} \left[(k_2 - 1) + k_2 \left\{ e_0 + \left(\frac{1}{2} - \alpha_6 \right) (e_1 + e_o e_1) + \left(\frac{\alpha_6}{2} - \frac{1}{8} \right) e_1^2 \right\} \right]$$

Bias and MSE of t_{p2} are

$$\begin{aligned}
 Bias(t_{p_2}) &= \bar{Y} \left[(k_2 - 1) + \lambda k_2 \left\{ \left(\frac{1}{2} - \alpha_6 \right) \rho C_y C_x + \left(\frac{\alpha_6}{2} - \frac{1}{8} \right) C_x^2 \right\} \right] \\
 MSE(t_{p_2}) &= \bar{Y}^2 (k_2 - 1)^2 + \lambda \bar{Y}^2 k_2^2 \left[C_y^2 + \left(\frac{1}{2} - \alpha_6 \right)^2 C_x^2 + 2 \left(\frac{1}{2} - \alpha_6 \right) \rho C_y C_x \right] \\
 &\quad + 2 \lambda \bar{Y}^2 k_2 (k_2 - 1) \left[\left(\frac{\alpha_6}{2} - \frac{1}{8} \right) C_x^2 + \left(\frac{1}{2} - \alpha_6 \right) \rho C_y C_x \right] \quad (28)
 \end{aligned}$$

Minimizing (28) w.r.t α_6 and k_2 , we get the following possible solution sets

Solution set I

$$k_2 = 0 \quad \text{and} \quad \alpha_6 = \frac{4\lambda\rho^2 C_x C_y - \lambda C_x^2 + 8}{4\lambda C_x (2\rho C_y - C_x)}$$

Solution set II

$$\begin{aligned}
 k_2 &= \frac{1}{8} \frac{16\lambda\rho^2 C_y^2 C_x^2 - 12\lambda\rho C_y C_x + \lambda C_x^2 - 8}{4\lambda\rho^2 C_y^2 - 2\lambda\rho C_y C_x - \lambda C_y^2 - 1} \\
 \alpha_6 &= \frac{2(4\lambda\rho^2 C_x C_y^2 - 3\lambda\rho C_y C_x^2 + 4\lambda\rho C_y^3 - 2\lambda C_x C_y^2 - 4\rho C_y - 2C_x)}{C_x (16\lambda\rho^2 C_y^2 - 12\lambda\rho C_y C_x + \lambda C_x^2 - 8)}
 \end{aligned}$$

By substituting the solution set II, we get the following minimum Bias and MSE

$$\begin{aligned}
 Bias(t_{p_2}) &= \frac{\lambda \bar{Y} (16 \lambda \rho^2 C_y^2 C_x^2 + 64 \lambda \rho^2 C_y^4 - 16 \lambda \rho C_y C_x^3 - \lambda C_x^4 - 16 \lambda C_y^2 C_x^2 - 192 \rho^2 C_y^2 + 64 C_y^2)}{64 (4 \lambda \rho^2 C_y^2 - 2 \lambda \rho C_y C_x - \lambda C_y^2 - 1)} \\
 MSE(t_{p_2}) &= \frac{\lambda \bar{Y}^2 (16 \lambda \rho^2 C_y^2 C_x^2 - 64 \lambda \rho^2 C_y^4 - 8 \lambda \rho C_y C_x^3 + 64 \lambda \rho C_x C_y^3 - \lambda C_x^4 - 16 \lambda C_y^2 C_x^2 - 64 \rho^2 C_y^2 + 64 C_y^2)}{64 (1 - 4 \lambda \rho^2 C_y^2 + 2 \lambda \rho C_y C_x + \lambda C_y^2)} \quad (29)
 \end{aligned}$$

Efficiency Comparison

Now we will compare the minimum MSE of the proposed class of estimators with other competing estimators discussed above in section 3 and the situations under which proposed estimators performs better than other estimators, are also given.

Condition (i). Comparing Eq. (2) and Eq. (27)

$$MSE(t_{p_1}) - Var(t_n) < 0, \text{ if}$$

$$\frac{\lambda C_x^6 + 4\lambda C_x^4 C_y^2 + 4\lambda C_x^2 C_y^4 + 2\lambda C_x^4 C_{yx} + 4\lambda C_x^2 C_y^2 C_{yx} + 4C_{yx}^2}{4\lambda C_x^2 C_{yx}^2 + 12\lambda C_y^2 C_{yx}^2} > 1$$

Condition (ii). Comparing Eq. (2) and Eq. (29)

$MSE(t_{p2}) - Var(t_n) < 0$, if

$$\frac{\lambda C_x^6 + 16\lambda C_x^4 C_y^2 + 64\lambda C_x^2 C_y^4 + 8\lambda C_x^4 C_{yx} + 64\lambda C_x^2 C_y^2 C_{yx} + 64C_{yx}^2}{16\lambda C_x^2 C_{yx}^2 + 192\lambda C_y^2 C_{yx}^2} > 1$$

Condition (iii). Comparing Eq. (4) and Eq. (27)

$MSE(t_{p1}) - MSE(t_r) < 0$, if

$$\frac{\lambda C_x^6 + 2\lambda C_x^4 C_y^2 + \lambda C_x^2 C_y^4 + \lambda C_x^4 C_{yx} + 8\lambda C_{yx}^3 + C_x^4 + C_{yx}^2}{\lambda C_y^2 C_x^2 C_{yx} + 9\lambda C_x^2 C_{yx}^2 + 3\lambda C_y^2 C_{yx}^2 + 2C_x^2 C_{yx}} > 1$$

Condition (iv). Comparing Eq. (4) and Eq. (29)

$MSE(t_{p2}) - MSE(t_r) < 0$, if

$$\frac{\lambda C_x^6 + 80\lambda C_x^4 C_y^2 + 64\lambda C_x^2 C_y^4 + 136\lambda C_x^4 C_{yx} + 512\lambda C_{yx}^3 + 64C_x^4 + 64C_{yx}^2}{64\lambda C_y^2 C_x^2 C_{yx} + 528\lambda C_x^2 C_{yx}^2 + 192\lambda C_y^2 C_{yx}^2 + 128C_x^2 C_{yx}} > 1$$

Condition (v). Comparing Eq. (8) and Eq. (27)

$MSE(t_{p1}) - MSE(t_r) < 0$, if

$$\frac{\lambda(3\theta_1^2 + 1)C_x^6 + \left[(40\theta_1 C_y^2 + 80\theta_1 C_{yx} + 4C_y^2 + 2C_{yx})\lambda + 40\theta_1 \right] C_x^4 + 4\lambda C_y^2 (C_y^2 + C_{yx}) C_x^2 + 32\lambda\theta_1 C_{yx}^3 + 4C_{yx}^2}{16 \left[\left\{ \frac{3}{8} \lambda \theta_1 C_x^4 + \left\{ \left(\theta_1 + \frac{1}{2} \right) 2C_{yx} + \frac{1}{2} \theta_1 C_y^2 \right\} \lambda + \frac{1}{2} \theta_1 \right\} C_x^2 + \frac{3}{4} \lambda C_y^2 C_{yx} \right] C_{yx}} > 1$$

Condition (vi). Comparing Eq. (8) and Eq. (29)

$MSE(t_{p2}) - MSE(t_{q1}) < 0$

$$\frac{\lambda C_x^4 + 128 \left(\theta_1^2 + \frac{1}{16} \right) \lambda \rho C_y C_x^3 + \left[(64\theta_1^2 + 16) \lambda C_y^2 + 64\theta_1^2 \right] C_x^2 + 512 \left(\theta_1^2 + \frac{1}{8} \right) \lambda \rho C_x C_y^3 + 64\lambda C_y^4 + 64\rho^2 C_y^2}{256 \left[\frac{3}{4} \lambda \rho C_y^3 + \frac{1}{2} \lambda \theta_1 C_y^2 C_x + \lambda \rho C_x^2 \left(\theta_1^2 + \theta_1 + \frac{1}{16} \right) C_y + \frac{1}{2} \theta_1 C_x \right] \rho C_y} > 1$$

Condition (vii). Comparing Eq. (10) and Eq. (27)

$MSE(t_{p1}) - MSE(t_{exp}) < 0$, if

$$\frac{7\lambda C_x^6 + 20\lambda C_x^4 C_y^2 + 16\lambda C_x^2 C_y^4 + 4\lambda C_x^4 C_{yx} + 64\lambda C_{yx}^3 + 4C_x^4 + 16C_{yx}^2}{64\lambda C_x^2 C_{yx}^2 + 48\lambda C_y^2 C_{yx}^2 + 16C_x^2 C_{yx}} > 1$$

Condition (viii). Comparing Eq. (10) and Eq. (29)

$$MSE(t_{p2}) - MSE(t_{exp}) < 0$$

$$\frac{256\lambda\rho^3 C_x C_y^3 + 40\lambda\rho C_x^3 C_y + \lambda C_x^4 + 32\lambda C_x^2 C_y^2 + 64\lambda C_y^4 + 64\rho^2 C_y^2 + 16C_x^2}{208\lambda\rho^2 C_x^2 C_y^2 + 192\lambda\rho^2 C_y^4 + 64\lambda\rho C_x C_y} > 1$$

Condition (ix). Comparing Eq. (16) and Eq. (27)

$$MSE(t_{p1}) - MSE(t_{is}) < 0, \text{ if}$$

$$\frac{\lambda \left[C_x^4 + (2C_y^2 + C_{yx}) C_x^2 - 4C_{yx}^2 \right]^2}{\theta_2^2 C_x^4 \left[3\lambda C_x^4 + C_x^2 \left\{ 4 + \lambda (4C_y^2 + 8C_{yx}) \right\} - 16\lambda C_{yx}^2 \right]} > 1$$

Condition (x). Comparing Eq. (16) and Eq. (29)

$$MSE(t_{p2}) - MSE(t_{is}) < 0, \text{ if}$$

$$\frac{\lambda \left[16\rho^2 C_y^4 - 4\rho C_y C_x - C_x^2 - 8C_y^2 \right]^2}{\theta_2^2 C_x^4 \left[-64 + \lambda C_y^2 (256\rho^2 - 64) - 128\lambda\rho C_y C_x \right]} > 1$$

Condition (xi). Comparing Eq. (18) and Eq. (27)

$$MSE(t_{p1}) - MSE(t_k) < 0, \text{ if}$$

$$\frac{\lambda \left[C_x^4 + (2C_y^2 + C_{yx}) C_x^2 - 4C_{yx}^2 \right]^2}{\theta_3^2 C_x^4 \left[3\lambda C_x^4 + C_x^2 \left\{ 4 + \lambda (4C_y^2 + 8C_{yx}) \right\} - 16\lambda C_{yx}^2 \right]} > 1$$

Condition (xii). Comparing Eq. (18) and Eq. (29)

$$MSE(t_{p2}) - MSE(t_k) < 0$$

$$\frac{\lambda \left[16\rho^2 C_y^4 - 4\rho C_y C_x - C_x^2 - 8C_y^2 \right]^2}{\theta_3^2 C_x^4 \left[-64 + \lambda C_y^2 (256\rho^2 - 64) - 128\lambda\rho C_y C_x \right]} > 1$$

Condition (xiii). Comparing Eq. (20) and Eq. (27)

$$MSE(t_{p1}) - MSE(t_{reg}) < 0$$

$$\frac{(4\rho^2 C_y^2 - \rho C_y C_x - C_x^2 - 2C_y^2)^2}{\lambda C_y^2 (16\rho^2 - 4) - 8\lambda \rho C_y C_x - 3\lambda C_x^2 - 4} < 0$$

Condition (xiv). Comparing Eq. (20) and Eq. (29)

$$MSE(t_{p_2}) - MSE(t_{reg}) < 0$$

$$\frac{(4\rho^2 C_y^2 - \rho C_y C_x - C_x^2 - 2C_y^2)^2}{\lambda C_y^2 (16\rho^2 - 4) - 8\lambda \rho C_y C_x - 3\lambda C_x^2 - 4} < 0$$

Numerical Study

Now the theoretical results of the proposed class of estimators are experienced against the existing estimators for the following populations. The percentage relative efficiency (PRE) of the estimators is computed using the succeeding formula

$$PRE = \frac{Var(t_n)}{MSE(t_i)} \times 100$$

Population I [Source: Fisher (1936)]

The study variable and auxiliary variable are allocated as;

Y= Petal length in cm and X= Petal width in cm

The summary statistics of the above data is as follows;

$$\begin{array}{llllll} N = 150 & n = 15 & \bar{Y} = 1.20 & \bar{X} = 3.76 & C_y = 0.64 & C_x = 0.47 \quad S_{yx} = 1.30 \\ \rho = 0.96 & Md = 4.35 & \beta_2 = 1.60 & Q_1 = 1.60 & & \end{array}$$

Population II [Source: AMIS (2014)]

The study variable and auxiliary variable are allocated as;

Y= Production of maze in “000” tones and X= Area in “000” hectares

The summary statistics of the above data is as follows;

$$\begin{array}{llllll} N = 120 & n = 8 & \bar{Y} = 30 & \bar{X} = 8 & C_y = 2.95 & C_x = 1.97 \\ S_{yx} = 1303.32 & \rho = 0.85 & Md = 0.95 & \beta_2 = 11.58 & Q_1 = 0.0 & \end{array}$$

Population III [Source: Rudolf et al. (2006)]

The study variable and auxiliary variable are allocated as;

Y= Total irrigated area (in hectares) and X= Total number of Tractors

A Class of Estimator for Population Mean Under SRS

The summary statistics of the above data is as follows;

$$\begin{array}{llllll} N = 69 & n = 8 & \bar{Y} = 345.75 & \bar{X} = 21.23 & C_y = 0.59 & C_x = 0.23 \quad \rho = 0.90 \\ Md = 15 & S_{yx} = 4467.69 & \beta_2 = 6.41 & Q_1 = 10 & & \end{array}$$

Population IV [Source: Waugh (1995)]

The study variable and auxiliary variable are allocated as;

Y = Whole weight of an abalone and X = Length of an abalone

The summary statistics of the above data is as follows

$$\begin{array}{llllll} N = 4177 & n = 25 & \bar{Y} = 0.8287 & \bar{X} = 0.5240 & C_y = 0.5917 & C_x = 0.2292 \\ S_{yx} = 0.0545 & \rho = 0.93 & Md = 0.545 & \beta_2 = 0.063 & Q_1 = 0.45 & \end{array}$$

Population V [Source: Arora (2017)]

The study variable and auxiliary variable are allocated as

Y = Atmospheric concentrations of MEI and X = Atmospheric concentrations of CH₄

The summary statistics of the above data is

$$\begin{array}{llllll} N = 308 & n = 20 & \bar{Y} = 0.276 & \bar{X} = 1749.83 & C_y = 3.404 & C_x = 0.026 \quad S_{yx} = -4.559 \\ \rho = -0.11 & Md = 1764.04 & \beta_2 = 2.643 & Q_1 = 1722.18 & & \end{array}$$

Population VI [Source: Arora (2017)]

The study variable and auxiliary variable are allocated as;

Y = The mean stratospheric Aerosol optical depth at 550 nm and X = Atmospheric concentrations of CFC₁₂

The summary statistics of the above data is as follows

$$\begin{array}{llllll} N = 308 & n = 20 & \bar{Y} = 0.0167 & \bar{X} = 497.53 & C_y = 1.744 & C_x = 0.116 \quad S_{yx} = -0.410 \\ \rho = -0.244 & Md = 528.36 & \beta_2 = 3.111 & Q_1 = 472.411 & & \end{array}$$

Population VII [Source: Arora (2017)]

The study variable and auxiliary variable are allocated as;

Y = The mean stratospheric Aerosol optical depth at 550 nm and X = Atmospheric concentrations of nitrous oxide N₂O

The summary of the statistics is as follows

$$N = 308 \quad n = 15 \quad \bar{Y} = 0.0167 \quad \bar{X} = 312.39 \quad C_y = 1.744 \quad C_x = 0.017 \quad \rho = -0.244$$

$$Md = 311.507 \quad S_{yx} = -0.0536 \quad \beta_2 = 0.145 \quad Q_1 = 308.11$$

Population VIII [Source: Agrestic (2012)]

The study variable and auxiliary variable are allocated as;

Y = Total registered votes in a state 1996 and X = Total votes taken by Ross Perot

The summary statistics of the above data is

$$N = 67 \quad n = 8 \quad \bar{Y} = 79115.6 \quad \bar{X} = 7220.54 \quad C_y = 1.49 \quad C_x = 1.24 \quad \rho = 0.94$$

$$Md = 3739 \quad S_{yx} = 989089058 \quad \beta_2 = 5.98 \quad Q_1 = 1072.50$$

Table 1: PRE of the estimators

Estimator	Data1	Data2	Data3	Data4	Data5	Data6	Data7	Data8
t_n	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
t_r	806.94	322.70	525.78	230.80	100.16	102.89	100.67	741.18
t_{ql}	368.12	322.70	388.80	152.02	100.08	101.57	100.34	5986
t_{exp}	234.71	184.08	267.20	147.25	100.08	101.54	100.34	254.21
t_{ts}	526.58	156.16	2025	555.75	101.13	106.20	114.28	234.22
t_k	450.04	228.83	1509	6994	101.12	105.87	114.27	121.94
t_{reg}, t_s, t_{rp}	1368.01	361.33	530.05	694.97	101.13	106.32	114.28	801.95
t_{p1}	1406.85	370.21	545.56	7161	1754	127.84	132.76	802.20
t_{p2}	1521.76	440.51	532.22	718.69	1754	127.76	132.76	989.62

Conclusions

In this article we have proposed an efficient class of estimators for the estimation of finite population mean under SRS. All the compulsory conditions of the data, for which the proposed class of estimators produce better estimates than usual estimator, ratio, exponential, Al- Omari et al. (2009), Singh and Espejo (2003), Tailor and Sharma (2009), Kumar (2015), Linear regression and Singh et al. (2008) estimator, are derived in section 5. An empirical study based on eight real life populations is also carried out to verify the theoretical results.

From the above study and results shown in Table 1, it is easily evident that our suggested class of estimators at their optimal conditions will always perform better than other competing estimators

when the efficiency conditions, discussed in section 5 are fulfilled by the data under study. Due to the minimum MSE of the proposed class of estimator, it is strongly recommended to use the proposed class of estimators in practical situations for the precise estimation of population mean.

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Some conversions of Suggested estimator

k	α	Estimator
k	α	$t_{p1} = k \bar{y} \left[\alpha \left(\frac{\bar{X}}{\bar{x}} \right) + (1 - \alpha) \left(\frac{\bar{x}}{\bar{X}} \right) \right]$ Proposed estimator
1	α	$t_{rp} = \bar{y} \left[\alpha \left(\frac{\bar{X}}{\bar{x}} \right) + (1 - \alpha) \left(\frac{\bar{x}}{\bar{X}} \right) \right]$ Singh and Espejo (2003)
k	1	$t_{kaur} = k \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)$ Kaur (1986)
k	0	$t_{kaur} = k \bar{y} \left(\frac{\bar{x}}{\bar{X}} \right)$ Kaur (1986)
1	1	$t_r = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)$ Cochran (1940)
1	0	$t_p = \bar{y} \left(\frac{\bar{x}}{\bar{X}} \right)$ Robson (1967)

Some conversions of suggested estimator

k	α	Estimator
k	α	$t_{p2} = k \bar{y} \left[\alpha \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) + (1 - \alpha) \exp \left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right) \right]$ Proposed estimator
1	α	$t_5 = \bar{y} \left[\alpha \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) + (1 - \alpha) \exp \left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right) \right]$ Singh et al. (2008)
k	1	$t = k \bar{y} \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$
k	0	$t = k \bar{y} \exp \left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right)$
1	1	$t_{\text{exp}} = \bar{y} \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$ Bahl and Tuteja (1991)
1	0	$t_{\text{exp}} = \bar{y} \exp \left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right)$ Bahl and Tuteja (1991)