A Class of Estimator for Population Mean Under SRS

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Abstract
In this paper we propose a class of estimators for the estimation of finite population mean using the auxiliary information when SRS scheme is used. The expressions for the Bias and mean square error (MSE) of the existing and suggested class of estimators are derived up to first degree of approximation and the efficiency comparison of suggested class of estimators is made with other existing estimators, using both theoretically and numerically based on real population sets.

Key words: Auxiliary variable, Bias, Mean square error, relative efficiency

Notations and Symbols
Let we have a population of fixed size N such that \( U = \{U_1, U_2, \ldots, U_N\} \). Each \( U_i = (Y_i, X_i), i = 1,2,3,\ldots,N \), has a pair of values. Here Y is the variable under study which is correlated with the auxiliary variable X. let \( y = \{y_1, y_2, \ldots, K, y_n\} \) and \( x = \{x_1, x_2, \ldots, K, x_n\} \) be n sample selected by using simple random sampling without replacement (SRSWOR), then consider the following notations

\[
\bar{Y} = \frac{1}{N} \sum_{i=1}^{N} Y_i \quad \text{Population mean of the study variable Y}
\]

\[
\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i \quad \text{Population mean of the auxiliary variable X}
\]

\[
S_y^2 = \frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \bar{Y})^2 \quad \text{Population variance of the Study variable Y}
\]
\[ S_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})^2 \]  
Population variance of the auxiliary variable X

\[ S_{yx} = \frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \bar{Y})(X_i - \bar{X}) \]  
Population covariance among Y and X

\[ C_y = \frac{S_y}{\bar{Y}} \]  
Population CV of the study variable Y

\[ C_x = \frac{S_x}{\bar{X}} \]  
Population CV of the auxiliary variable X

\[ \rho = \frac{C_{yx}}{C_y C_x} \]  
Population Correlation coefficient between Y and X

\[ \beta_1 = \frac{\mu_3}{\mu_2^2} \]  
Coefficient of skewness of the auxiliary variable X

\[ \beta_2 = \frac{\mu_4}{\mu_2^2} \]  
Coefficient of kurtosis of the auxiliary variable X

Md  
Median of the auxiliary variable X

\[ \lambda = \left(1 - \frac{1}{N} \right) \]  
FPCF

The following useful error terms are considered to derive the expressions for the Bias and MSE of the estimators

\[ e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}} \quad e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}} \]  
Such that \( \bar{y} = \bar{Y} (1 + e_0) \) and \( \bar{x} = \bar{X} (1 + e_1) \)

The expected values of the above error terms are given as

\[ E(e_0) = 0 \quad E(e_1) = 0 \]

\[ E(e_0^2) = \lambda C_y^2 \quad E(e_1^2) = \lambda C_x^2 \quad E(e_0 e_1) = \lambda \rho C_y C_x \]

Where \( \rho \) is the coefficient of correlation between study variable Y and the auxiliary variable X.

**Existing Estimators**

Here we will discourse a number of existing estimators of the population mean available in literature. The expressions for the bias and MSE of each of these estimators up to the first order of approximation are also given.
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The usual mean estimator of the study variable is

\[ t_n = \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \]  

(1)

The variance of \( t_n \) is given by

\[ \text{Var}(t_n) = \lambda \bar{Y}^2 C_y^2 \]  

(2)

The ratio estimator is given by

\[ t_r = \bar{y} \left( \frac{\bar{X}}{\bar{x}} \right) \]  

(3)

The bias and MSE of \( t_r \) are given by

\[ \text{Bias}(t_r) = \lambda \bar{Y}^2 \left( C_y^2 - \rho C_y C_x \right) \]

and

\[ \text{MSE}(t_r) = \lambda \bar{Y}^2 \left( C_y^2 + C_x^2 - 2 \rho C_y C_x \right) \]  

(4)

In case of negatively correlation, the product estimator is given by

\[ t_p = \bar{y} \left( \frac{\bar{X}}{\bar{x}} \right) \]  

(5)

Bias and MSE of \( t_p \) are given by

\[ \text{Bias}(t_p) = \lambda \bar{Y} \rho C_y C_x \]

and

\[ \text{MSE}(t_p) = \lambda \bar{Y}^2 \left( C_y^2 + C_x^2 + 2 \rho C_y C_x \right) \]  

(6)

Many researchers modified the above estimators by using the further information of auxiliary variable like median, quartiles, coefficient of skewness and kurtosis etc. Al-Omari et al. (2009) proposed the following ratio type estimator, by using the known first quartile of the auxiliary variable

\[ t_{q1} = \bar{y} \left( \frac{\bar{X} + Q_1}{\bar{x} + Q_1} \right) \]  

(7)

The bias and MSE of \( t_{q1} \) are given by

\[ \text{Bias}(t_{q1}) = \lambda \bar{Y} \left( C_x^2 \theta_1^2 - \rho \theta_1 C_y C_x \right) \]
and \[ MSE (t_{q_1}) = \lambda \bar{Y}^2 \left( C_y^2 + C_x^2 \theta_1^2 - \rho \theta_1 C_y C_x \right) \] (8)

where \[ \theta_1 = \frac{\bar{X}}{\bar{X} + q_1} \]

Bahl and Tuteja (1991) proposed the following ratio type estimator

\[ t_{exp} = \bar{y} \exp \left( \frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \] (9)

Bias and MSE of \( t_{exp} \) are given by

\[ Bias (t_{exp}) = \lambda \bar{Y} \left( \frac{3}{8} C_y^2 - \frac{1}{2} \rho C_y C_x \right) \]

and \[ MSE (t_{exp}) = \lambda \bar{Y}^2 \left( C_y^2 + \frac{1}{4} C_x^2 - \rho C_y C_x \right) \] (10)

Bahl and Tuteja (1991) proposed the following product type estimator

\[ t_{exp2} = \bar{y} \exp \left( \frac{\bar{x} - \bar{X}}{\bar{X} + \bar{X}} \right) \] (11)

Bias and MSE of \( t_{exp2} \) are given by

\[ Bias (t_{exp2}) = \lambda \bar{Y} \left( -\frac{1}{8} C_x^2 + \frac{1}{2} \rho C_y C_x \right) \]

and \[ MSE (t_{exp2}) = \lambda \bar{Y}^2 \left( C_y^2 + \frac{1}{4} C_x^2 + \rho C_y C_x \right) \] (12)

By combining ratio and product estimator, Singh and Espejo (2003) proposed the following estimator

\[ t_{rp} = \bar{y} \left[ \alpha_i \left( \frac{\bar{X}}{\bar{X}} \right) + (1 - \alpha_i) \left( \frac{\bar{x}}{\bar{X}} \right) \right] \] (13)

Where \( \alpha_i \) is a constant with the optimum value as given by

\[ \alpha_{opt} = \frac{1}{2} + \frac{1}{2} \frac{\rho C_y}{C_x} \]
Bias and MSE of $t_{rp}$ are given as

$$\text{Bias} (t_{rp}) = \lambda \bar{Y} \left( \frac{1}{2} + \frac{1}{2} \rho \frac{C_y}{C_x} C_x^2 - \rho^2 C_y^2 \right)$$

and

$$\text{MSE} (t_{rp}) = \lambda \bar{Y}^2 C_y^2 \left( 1 - \rho^2 \right)$$

(14)

Which is equivalent to the min MSE of the linear regression estimator

Tailor and Sharma (2009) reformed the Singh and Espejo (2003) estimator by using the known C.V and coefficient of kurtosis of the auxiliary variable as

$$t_{ts} = \bar{Y} \left[ \alpha_2 \left( \frac{C_s \bar{X} + \beta_2}{C_s \bar{X} + \beta_2} \right) + (1 - \alpha_2) \left( \frac{C_s \bar{X} + \beta_2}{C_s \bar{X} + \beta_2} \right) \right]$$

(15)

where $\alpha_2$ is constant with the following optimum value

$$\alpha_{opt}^2 = \frac{1}{2} \frac{\rho C_y}{\theta_2 C_x}$$

Bias and MSE of $t_{ts}$ are given by

$$\text{Bias} (t_{ts}) = - \lambda \bar{Y} \rho C_y \left( \rho C_y - \frac{3}{2} \theta_2 C_x^2 \right)$$

$$\text{MSE} (t_{ts}) = \lambda \bar{Y}^2 C_y^2 \left[ \left( 1 - \rho^2 \right) C_y^2 - C_x^2 \theta_2^2 \right]$$

where

$$\theta_2 = \frac{C_s \bar{X}}{C_s \bar{X} + \beta_2}$$

The min MSE of $t_{ts}$ for the optimum value of $\alpha_2$ is equivalent to the min MSE of linear regression estimator i.e.

$$\text{Min MSE} (t_{ts}) = \lambda \bar{Y}^2 C_y^2 \left( 1 - \rho^2 \right)$$

Kumar (2015) used the known median of auxiliary variable and reformed Singh and Espejo (2003) estimator as

$$t_k = \bar{Y} \left[ \alpha_3 \left( \frac{\bar{X} + Md}{\bar{X} + Md} \right) + (1 - \alpha_3) \left( \frac{\bar{X} + Md}{\bar{X} + Md} \right) \right]$$

(17)
where $\alpha_3$ is constant with the following optimum value

$$\alpha_{3,\text{opt}} = \frac{1}{2} \frac{\rho C_y}{\theta_3 C_x}$$

Bias and MSE of $t_k$, are as below

$$Bias(t_k) = -\lambda \bar{Y} \rho C_y \left( \rho C_y - \frac{3}{2} \theta_3 C_x^2 \right)$$

$$MSE(t_k) = \lambda \bar{Y}^2 C_y^2 \left[ (1 - \rho^2) C_y^2 - C_x^2 \theta_3^2 \right] \quad (18)$$

where

$$\theta_3 = \frac{\bar{X}}{\bar{X} + Md}$$

The min MSE of $t_k$ for the optimum value of $\alpha_3$ is equivalent to the min MSE of linear regression estimator i.e

$$Min \ MSE(t_k) = \lambda \bar{Y}^2 C_y^2 (1 - \rho^2)$$

The very known linear regression estimator is

$$t_{\text{reg}} = \bar{y} + b_{ys} \left( \bar{X} - \bar{x} \right) \quad (19)$$

Where $b_{ys} = \frac{s_{xy}}{s_x^2}$ is coefficient of sample regression. The bias and MSE of $t_{\text{reg}}$ are given by

$$Bias(t_{\text{reg}}) = -\lambda \beta S_x \left( \frac{\lambda_{12}}{\rho} - \lambda_{03} \right)$$

$$\text{min MSE}(t_{\text{reg}}) = \lambda \bar{Y}^2 C_y^2 (1 - \rho^2) \quad (20)$$

Singh et al. (2008) suggested a ratio-cum-product exponential type estimator as

$$t_s = \bar{y} \left[ \alpha_4 \exp \left( \frac{\bar{X} - \bar{y}}{\bar{X} + \bar{y}} \right) + (1 - \alpha_4) \exp \left( \frac{\bar{X} - \bar{X}}{\bar{X} + \bar{X}} \right) \right] \quad (21)$$

Where $\alpha_4$ constant whose optimum value is given by
\[ \alpha_{4y} = \frac{1}{2} + \frac{\rho C_y}{C_x} \]

The bias and MSE of \( t_y \) are given by

\[
\text{Bias}(t_y) = -\frac{1}{8} \lambda \bar{Y} (8 \rho^2 C_y^2 - 4 \rho C_y C_x - C_x^2)
\]

\[
\text{Min MSE}(t_y) = \lambda \bar{Y}^2 C_y C_x (1 - \rho^2)
\] (22)

Which is equivalent to the min MSE of linear Regression estimator

**Proposed Class of Estimators**

Our proposed class of estimators is in the line of Searls (1964). Here we propose the following class of estimators

\[
t_{p1} = k_1 \bar{y} \left[ \alpha_5 \left( \frac{\bar{X}}{k_1} \right) + (1 - \alpha_3) \left( \frac{\bar{X}}{k_1} \right) \right]
\] (23)

\[
t_{p2} = k_2 \bar{y} \left[ \alpha_6 \exp \left( \frac{\bar{X} - \bar{X}}{k_2} \right) + (1 - \alpha_6) \exp \left( \frac{\bar{X} - \bar{X}}{k_2} \right) \right]
\] (24)

where \( k_1, k_2, \alpha_5 \) and \( \alpha_6 \) are constants

Now solving equation (23)

\[
t_{p1} = k_1 \bar{Y} (1 + e_0) \left[ \alpha_5 (1 + e_1)^{-1} + (1 - \alpha_3) (1 + e_1) \right]
\]

\[
t_{p1} = k_1 \bar{Y} \left[1 + e_0 + (1 - 2 \alpha_5) (e_1 + e_0 e_1) + \alpha_4 e_1^2 \right]
\]

\[
t_{p1} - \bar{Y} = k_1 \bar{Y} \left[ (k_1 - 1) + k_1 \left\{ e_0 + (1 - 2 \alpha_5) (e_1 + e_0 e_1) + \alpha_4 e_1^2 \right\} \right]
\] (25)

Bias and MSE of \( t_{p1} \) are given by

\[
\text{Bias}(t_{p1}) = \bar{Y} E \left[ (k_1 - 1) + \lambda k_1 \left\{ (1 - 2 \alpha_5) \rho C_y C_x + \alpha_5 C_x^2 \right\} \right]
\]

and

\[
\text{MSE}(t_{p1}) = \bar{Y}^2 (k_1 - 1)^2 + \lambda \bar{Y}^2 k_1^2 \left\{ C_x^2 + (1 - 2 \alpha_5)^2 C_x^2 + 2 (1 - 2 \alpha_5) \rho C_y C_x \right\} + 2 \lambda \bar{Y}^2 k_1 (k_1 - 1)
\]

\[
\left\{ \alpha_4 C_x^2 + (1 - 2 \alpha_5) \rho C_y C_x \right\}
\] (26)
Now minimizing (26) w.r.t $\alpha_5$ and $k_1$, up to zero, we get the following two solution set:

**Solution set I**

$$k_1 = 0 \quad \text{and} \quad \alpha_5 = \frac{\lambda \rho C_y C_x + 1}{\lambda C_x (2 \rho C_y - C_x)}$$

**Solution set II**

$$k_1 = \frac{8 \lambda \rho^2 C_y^2 - 6 \lambda \rho C_y C_x - \lambda C_x^2 - 4}{16 \lambda \rho^2 C_y^2 - 8 \lambda \rho C_y C_x - 3 \lambda C_x^2 - 4 \lambda C_y^2 - 4}$$

and

$$\alpha_5 = \frac{4 \lambda \rho^2 C_x C_y^2 - 3 \lambda \rho C_y C_x^2 + 2 \lambda \rho C_y^3 - \lambda C_x C_y^2 - 2 \rho C_x - 2 C_x}{C_x (8 \lambda \rho^2 C_y^2 - 6 \lambda \rho C_y C_x - \lambda C_x^2 - 4)}$$

by substituting the values of solution set II, we get minimum bias and MSE as

$$\text{Bias}(t_{p1}) = \frac{\lambda \bar{Y} (4 \lambda \rho^2 C_x C_y^2 - 4 \lambda \rho^2 C_y^4 - 2 \lambda \rho C_y C_x^3 + 4 \lambda \rho C_y C_x - \lambda C_x C_y^2 - 2 \rho C_x - 2 C_x)}{4 + 4 \lambda \rho^2 C_y^2 + 3 \lambda \rho C_y C_x - 16 \lambda \rho^2 C_y^2}$$

and

$$\text{MSE}(t_{p1}) = \frac{\lambda \bar{Y}^2 (4 \lambda \rho^2 C_x^3 C_y^2 - 4 \lambda \rho^2 C_y^4 - 2 \lambda \rho C_y C_x^3 + 4 \lambda \rho C_y C_x^2 - \lambda C_x^2 C_y^2 - 4 \rho^2 C_x^2 + 4 C_y^2)}{4 + 4 \lambda \rho^2 C_y^2 + 3 \lambda \rho C_x C_y^2 + 8 \lambda \rho C_y C_x^2 - 16 \lambda \rho^2 C_y^2}$$

(27)

Now solving (24)

$$t_{p2} = k_2 \bar{y} \left[ \alpha_6 \exp \left( -\frac{e_1}{2} \left( 1 + \frac{e_1}{2} \right)^{-1} \right) + (1 - \alpha_6) \exp \left( \frac{e_1}{2} \left( 1 + \frac{e_1}{2} \right)^{-1} \right) \right]$$

$$t_{p2} = k_2 \bar{y} \left[ \alpha_6 \exp \left( -\frac{e_1}{4} \right) + (1 - \alpha_6) \exp \left( \frac{e_1}{4} \right) \right]$$

$$t_{p2} = k_2 \bar{y} \left[ \alpha_6 \left( 1 - \frac{e_1}{2} + \frac{e_1^2}{4} + \frac{e_1^2}{8} \right) + (1 - \alpha_6) \left( 1 + \frac{e_1}{2} + \frac{e_1^2}{4} + \frac{e_1^2}{8} \right) \right]$$

$$t_{p2} = -\bar{y} \left[ (k_2 - 1) + k_2 \left\{ e_0 + \left( \frac{1}{2} - \alpha_6 \right) (e_1 + e_1^2) + 2 \left( \alpha_6 - \frac{1}{8} \right) e_1^2 \right\} \right]$$

Bias and MSE of $t_{p2}$ are
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\[ \text{Bias}(t_{p_2}) = \bar{Y} \left[ (k_2 - 1) + \lambda k_2 \left\{ \left( \frac{1}{2} - \alpha_6 \right) \rho C_y C_x + \left( \frac{\alpha_6}{2} - \frac{1}{8} \right) C_x^2 \right\} \right] \]

\[ \text{MSE}(t_{p_2}) = \bar{Y}^2 (k_2 - 1)^2 + \lambda \bar{Y}^2 k_2 \left[ C_y^2 + \left( \frac{1}{2} - \alpha_6 \right)^2 C_x^2 + 2 \left( \frac{1}{2} - \alpha_6 \right) \rho C_y C_x \right] \]

\[ + 2\lambda \bar{Y}^2 k_2 (k_2 - 1) \left[ \left( \frac{\alpha_6}{2} - \frac{1}{8} \right) C_x^2 + \left( \frac{1}{2} - \alpha_6 \right) \rho C_y C_x \right] \quad (28) \]

Minimizing (28) w.r.t \( \alpha_6 \) and \( k_2 \), we get the following possible solution sets

Solution set I

\[ k_2 = 0 \quad \text{and} \quad \alpha_6 = \frac{4\lambda \rho 2C_y C_x - \lambda C_x^2 + 8}{4\lambda C_x (2\rho C_y - C_x)} \]

Solution set II

\[ k_2 = \frac{1}{8} \frac{16\lambda \rho^2 C_y^2 - 12\lambda \rho C_y C_x + \lambda C_x^2 - 8}{4\lambda \rho^2 C_y^2 - 2\lambda \rho C_y C_x - \lambda C_x^2 - 1} \]

\[ \alpha_6 = \frac{2(4\lambda \rho^2 C_y^2 - 3\lambda \rho C_y C_x^2 + 4\lambda \rho C_y^3 - 2\lambda C_y C_x^2 - 4\rho C_y - 2C_x)}{C_x \left( 16\lambda \rho^2 C_y^2 - 12\lambda \rho C_y C_x + \lambda C_x^2 - 8 \right)} \]

By substituting the solution set II, we get the following minimum Bias and MSE

\[ \text{Bias}(t_{p_2}) = \frac{\lambda \bar{Y} (16 \lambda \rho^2 C_y^2 C_x^2 + 64 \lambda \rho^2 C_y^4 - 16 \lambda \rho C_y C_x^3 - 16 \lambda C_y^2 C_x^2 - 192 \rho^2 C_y^2 + 64 C_y^2)}{64 (4 \lambda \rho^2 C_y^2 - 2 \lambda \rho C_y C_x - \lambda C_y^2 - 1)} \]

\[ \text{MSE}(t_{p_2}) = \frac{\lambda \bar{Y}^2 (16 \lambda \rho^2 C_y^2 C_x^2 - 64 \lambda \rho^2 C_y^4 - 8 \lambda \rho C_y C_x^3 + 64\lambda \rho C_y C_x^3 - \lambda C_y^4 - 16 \lambda C_y^2 C_x^2 - 64 \rho^2 C_y^2 + 64 C_y^2)}{64 (1 - 4 \lambda \rho^2 C_y^2 + 2 \lambda \rho C_y C_x + \lambda C_y^2)} \]  

(29)

**Efficiency Comparison**

Now we will compare the minimum MSE of the proposed class of estimators with other competing estimators discussed above in section 3 and the situations under which proposed estimators perform better than other estimators, are also given.

Condition (i). Comparing Eq. (2) and Eq. (27)

\[ \text{MSE}(t_{p_1}) - \text{Var}(t_{p}) < 0, \text{ if} \]

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\[
\frac{\lambda C_x^6 + 4\lambda C_x^4C_y^2 + 4\lambda C_x^2C_y^4 + 2\lambda C_x^4C_{xy} + 4\lambda C_x^2C_y^2C_{xy} + 4C_x^2}{4\lambda C_x^2C_{xy}^2 + 12\lambda C_y^2C_{xy}^2} > 1
\]

Condition (ii). Comparing Eq. (2) and Eq. (29)
\[\text{MSE}(t_{p2}) - \text{Var}(t_p) < 0, \text{ if} \]
\[
\frac{\lambda C_x^6 + 16\lambda C_x^4C_y^2 + 64\lambda C_x^2C_y^4 + 8\lambda C_x^4C_{xy} + 64\lambda C_x^2C_y^2C_{xy} + 64C_x^2}{16\lambda C_x^2C_{xy}^2 + 192\lambda C_y^2C_{xy}^2} > 1
\]

Condition (iii). Comparing Eq. (4) and Eq. (27)
\[\text{MSE}(t_{p2}) - \text{MSE}(t_r) < 0, \text{ if} \]
\[
\frac{\lambda C_x^6 + 2\lambda C_x^4C_y^2 + \lambda C_x^2C_y^4 + \lambda C_x^4C_{xy} + 8\lambda C_x^2C_y^2C_{xy} + C_x^4 + C_{xy}^2}{\lambda C_y^2C_xC_{xy}^2 + 9\lambda C_x^2C_y^2 + 3\lambda C_y^2C_{xy}^2 + 2C_x^2C_{xy}} > 1
\]

Condition (iv). Comparing Eq. (4) and Eq. (29)
\[\text{MSE}(t_{p2}) - \text{MSE}(t_r) < 0, \text{ if} \]
\[
\frac{\lambda C_x^6 + 80\lambda C_x^4C_y^2 + 64\lambda C_x^2C_y^4 + 136\lambda C_x^4C_{xy} + 512\lambda C_x^3C_{xy} + 64\lambda C_x^2C_{xy}^2}{64\lambda C_y^2C_xC_{xy}^2 + 528\lambda C_x^2C_y^2C_{xy} + 192\lambda C_y^2C_{xy}^2 + 128C_x^2C_{xy}} > 1
\]

Condition (v). Comparing Eq. (8) and Eq. (27)
\[\text{MSE}(t_{p1}) - \text{MSE}(t_r) < 0, \text{ if} \]
\[
\lambda \left(3\theta_1^2 + 1\right)C_x^6 + \left[\left(4\theta_1^2 + 8\theta_1C_{xy} + 4C_{xy}^2 + 2C_{xy}\right)C_x^4 + 4\lambda \theta_1 \left(C_y^2 + C_{xy}\right)C_x^2 + 32\lambda \theta_1 C_{xy}^2 + 4C_{xy}^2\right] > 1
\]

Condition (vi). Comparing Eq. (8) and Eq. (29)
\[\text{MSE}(t_{p2}) - \text{MSE}(t_{q1}) < 0, \text{ if} \]
\[
\frac{\lambda C_x^4 + 128\left(\theta_x^2 + \frac{1}{16}\right)\lambda \rho C_{xy}^3 + \left(64\theta_x^2 + 16\right)\lambda C_{xy}^2 + 64\lambda C_{xy}^2}{256\left[3\frac{3}{4}\lambda \rho C_{xy}^3 + \frac{1}{2}\lambda \theta_x C_{xy}^2 + \lambda \rho C_{xy}^2 \left(\theta_x^2 + \theta_1 + \frac{1}{16}\right)C_{xy} + \frac{1}{2}\theta_1 C_{xy}\right] \rho C_{xy}} > 1
\]

Condition (vii). Comparing Eq. (10) and Eq. (27)
\[\text{MSE}(t_{p1}) - \text{MSE}(t_{exp}) < 0, \text{ if} \]
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\[ \frac{7\lambda C_x^6 + 20\lambda C_x^4 C_y^2 + 16\lambda C_x^2 C_y^4 + 4\lambda C_x^4 C_{xy} + 64\lambda C_{xy}^3 + 4 C_y^4 + 16 C_{xy}^2}{64\lambda C_x^2 C_{xy}^2 + 48\lambda C_y^2 C_{xy}^2 + 16 C_x^2 C_{xy}^2} > 1 \]

Condition (viii). Comparing Eq. (10) and Eq. (29)

\[ MSE(t_{p2}) - MSE(t_{exp}) < 0 \]

\[ \frac{256\lambda \rho^2 C_x^6 C_y^3 + 40\lambda \rho C_x^4 C_y^2 + 8\lambda C_x^4 C_y^2 + 64\lambda C_y^4 + 64\rho^2 C_y^2 + 16 C_x^2}{208\lambda \rho^2 C_x^2 C_y^2 + 192\lambda \rho C_y C_y^2 + 64\lambda \rho C_y C_y^2} > 1 \]

Condition (ix). Comparing Eq. (16) and Eq. (27)

\[ MSE(t_{p3}) - MSE(t_{rs}) < 0 \]

\[ \frac{\lambda \left[ C_x^4 + (2C_y^2 + C_{xy})C_y^2 - 4C_{xy}^2 \right]^2}{\theta^2 C_x \left[ 3\lambda C_x^4 + C_x^2 \left( 4 + \lambda \left( 4C_y^2 + 8C_{xy} \right) \right) \right] - 16\lambda C_{xy}^2} > 1 \]

Condition (x). Comparing Eq. (16) and Eq. (29)

\[ MSE(t_{p2}) - MSE(t_{rs}) < 0 \], if

\[ \frac{\lambda \left[ 16\rho^2 C_y^4 - 4\rho C_y C_x - C_x^2 - 8C_y^2 \right]^2}{\theta^2 C_x \left[ -64 + \lambda C_y^2 \left( 256\rho^2 - 64 \right) - 128\lambda \rho C_y C_x \right]} > 1 \]

Condition (xi). Comparing Eq. (18) and Eq. (27)

\[ MSE(t_{p3}) - MSE(t_{rs}) < 0 \], if

\[ \frac{\lambda \left[ C_x^4 + (2C_y^2 + C_{xy})C_y^2 - 4C_{xy}^2 \right]^2}{\theta^2 C_x \left[ 3\lambda C_x^4 + C_x^2 \left( 4 + \lambda \left( 4C_y^2 + 8C_{xy} \right) \right) \right] - 16\lambda C_{xy}^2} > 1 \]

Condition (xii). Comparing Eq. (18) and Eq. (29)

\[ MSE(t_{p2}) - MSE(t_{rs}) < 0 \]

\[ \frac{\lambda \left[ 16\rho^2 C_y^4 - 4\rho C_y C_x - C_x^2 - 8C_y^2 \right]^2}{\theta^2 C_x \left[ -64 + \lambda C_y^2 \left( 256\rho^2 - 64 \right) - 128\lambda \rho C_y C_x \right]} > 1 \]

Condition (xiii). Comparing Eq. (20) and Eq. (27)

\[ MSE(t_{p3}) - MSE(t_{reg}) < 0 \]
\[
\frac{\left(4\rho^2C_y^2 - \rho C_C C_C - C_C^2 - 2C_y^2\right)}{\lambda C_y^2 \left(16\rho^2 - 4\right) - 8\lambda \rho C_C C_C - 3\lambda C_C^2 - 4} < 0
\]

Condition (xiv). Comparing Eq. (20) and Eq. (29)

\[
MSE(t_{p_2}) - MSE(t_{reg}) < 0
\]

Numerical Study

Now the theoretical results of the proposed class of estimators are experienced against the existing estimators for the following populations. The percentage relative efficiency (PRE) of the estimators is computed using the succeeding formula

\[
PRE = \frac{Var(t_x)}{MSE(t)} \times 100
\]

Population I [Source: Fisher (1936)]

The study variable and auxiliary variable are allocated as;

\(Y=\) Petal length in cm and \(X=\) Petal width in cm

The summary statistics of the above data is as follows;

\[
N = 150 \quad n = 15 \quad \bar{Y} = 1.20 \quad \bar{X} = 3.76 \quad C_y = 0.64 \quad C_x = 0.47 \quad S_{ys} = 1.30
\]

\(\rho = 0.96 \quad Md = 4.35 \quad \beta_2 = 1.60 \quad Q_1 = 1.60\)

Population II [Source: AMIS (2014)]

The study variable and auxiliary variable are allocated as;

\(Y=\) Production of maze in “000” tones and \(X=\) Area in “000” hectares

The summary statistics of the above data is as follows;

\[
N = 120 \quad n = 8 \quad \bar{Y} = 30 \quad \bar{X} = 8 \quad C_y = 2.95 \quad C_x = 1.97
\]

\(S_{ys} = 1303.32 \quad \rho = 0.85 \quad Md = 0.95 \quad \beta_2 = 11.58 \quad Q_1 = 0.0\)

Population III [Source: Rudalf et al. (2006)]

The study variable and auxiliary variable are allocated as;

\(Y=\) Total irrigated area (in hectares) and \(X=\) Total number of Tractors

20
A Class of Estimator for Population Mean Under SRS

The summary statistics of the above data is as follows;

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 69 )</td>
<td>( n = 8 )</td>
</tr>
<tr>
<td>( Md = 15 )</td>
<td>( S_{yx} = 4467.69 )</td>
</tr>
</tbody>
</table>

The study variable and auxiliary variable are allocated as;

\( Y = \) Whole weight of an abalone and \( X = \) Length of an abalone

The summary statistics of the above data is as follows

<table>
<thead>
<tr>
<th>Population V</th>
<th>Source: Arora (2017)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 4177 )</td>
<td>( n = 25 )</td>
</tr>
<tr>
<td>( S_{yx} = 0.0545 )</td>
<td>( \rho = 0.93 )</td>
</tr>
</tbody>
</table>

The study variable and auxiliary variable are allocated as

\( Y = \) Atmospheric concentrations of MEI and \( X = \) Atmospheric concentrations of CH4

The summary statistics of the above data is

<table>
<thead>
<tr>
<th>Population VI</th>
<th>Source: Arora (2017)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 308 )</td>
<td>( n = 20 )</td>
</tr>
<tr>
<td>( \rho = -0.11 )</td>
<td>( Md = 1764.04 )</td>
</tr>
</tbody>
</table>

The study variable and auxiliary variable are allocated as

\( Y = \) The mean stratospheric Aerosol optical depth at 550 nm and \( X = \) Atmospheric concentrations of CFC.12

The summary statistics of the above data is as follows

<table>
<thead>
<tr>
<th>Population VII</th>
<th>Source: Arora (2017)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 308 )</td>
<td>( n = 20 )</td>
</tr>
<tr>
<td>( \rho = -0.244 )</td>
<td>( Md = 528.36 )</td>
</tr>
</tbody>
</table>

The study variable and auxiliary variable are allocated as

\( Y = \) The mean stratospheric Aerosol optical depth at 550 nm and \( X = \) Atmospheric concentrations of nitrous oxide N2O

The summary of the statistics is as follows
Population VIII [Source: Agrestic (2012)]

The study variable and auxiliary variable are allocated as;

\( Y = \) Total registered votes in a state 1996 and \( X = \) Total votes taken by Ross Perot

The summary statistics of the above data is

\[
\begin{align*}
N &= 67 \quad n = 8 \\
\bar{Y} &= 79115.6 \quad \bar{X} = 7220.54 \quad \sigma_Y = 1.49 \quad \sigma_X = 1.24 \quad \rho = 0.94 \\
Md &= 3739 \quad S_{yx} = 989089058 \quad \beta_2 = 5.98 \quad Q_1 = 1072.50
\end{align*}
\]

### Table 1: PRE of the estimators

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Data1</th>
<th>Data2</th>
<th>Data3</th>
<th>Data4</th>
<th>Data5</th>
<th>Data6</th>
<th>Data7</th>
<th>Data8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_n )</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>806.94</td>
<td>322.70</td>
<td>525.78</td>
<td>230.80</td>
<td>100.16</td>
<td>102.89</td>
<td>100.67</td>
<td>741.18</td>
</tr>
<tr>
<td>( t_{ql} )</td>
<td>368.12</td>
<td>322.70</td>
<td>388.80</td>
<td>152.02</td>
<td>100.08</td>
<td>101.57</td>
<td>100.34</td>
<td>5986</td>
</tr>
<tr>
<td>( t_{exp} )</td>
<td>234.71</td>
<td>184.08</td>
<td>267.20</td>
<td>147.25</td>
<td>100.08</td>
<td>101.54</td>
<td>100.34</td>
<td>254.21</td>
</tr>
<tr>
<td>( t_{ts} )</td>
<td>526.58</td>
<td>156.16</td>
<td>2025</td>
<td>555.75</td>
<td>101.13</td>
<td>106.20</td>
<td>114.28</td>
<td>234.22</td>
</tr>
<tr>
<td>( t_k )</td>
<td>450.04</td>
<td>228.83</td>
<td>1509</td>
<td>6994</td>
<td>101.12</td>
<td>105.87</td>
<td>114.27</td>
<td>121.94</td>
</tr>
<tr>
<td>( t_{reg}, \bar{t}_x, \bar{t}_q )</td>
<td>1368.01</td>
<td>361.33</td>
<td>530.05</td>
<td>694.97</td>
<td>101.13</td>
<td>106.32</td>
<td>114.28</td>
<td>801.95</td>
</tr>
<tr>
<td>( t_{pl} )</td>
<td>1406.85</td>
<td>370.21</td>
<td>545.56</td>
<td>7161</td>
<td>1754</td>
<td>127.84</td>
<td>132.76</td>
<td>802.20</td>
</tr>
<tr>
<td>( t_{p2} )</td>
<td>1521.76</td>
<td>440.51</td>
<td>532.22</td>
<td>718.69</td>
<td>1754</td>
<td>127.76</td>
<td>132.76</td>
<td>989.62</td>
</tr>
</tbody>
</table>

### Conclusions

In this article we have proposed an efficient class of estimators for the estimation of finite population mean under SRS. All the compulsory conditions of the data, for which the proposed class of estimators produce better estimates than usual estimator, ratio, exponential, Al- Omari et al. (2009), Singh and Espejo (2003), Tailor and Sharma (2009), Kumar (2015), Linear regression and Singh et al. (2008) estimator, are derived in section 5. An empirical study based on eight real life populations is also carried out to verify the theoretical results.

From the above study and results shown in Table 1, it is easily evident that our suggested class of estimators at their optimal conditions will always perform better than other competing estimators.
A Class of Estimator for Population Mean Under SRS

when the efficiency conditions, discussed in section 5 are fulfilled by the data under study. Due to the minimum MSE of the proposed class of estimator, it is strongly recommended to use the proposed class of estimators in practical situations for the precise estimation of population mean.

Acknowledgments

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References


### Some conversions of Suggested estimator

<table>
<thead>
<tr>
<th>$k$</th>
<th>$a$</th>
<th>Estimator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$a$</td>
<td>$t_{pl} = k \bar{y} \left[ \alpha \left( \frac{\bar{X}}{\bar{x}} \right) + (1-\alpha) \left( \frac{\bar{x}}{X} \right) \right]$ Proposed estimator</td>
</tr>
<tr>
<td>1</td>
<td>$a$</td>
<td>$t_{rp} = \bar{y} \left[ \alpha \left( \frac{\bar{X}}{\bar{x}} \right) + (1-\alpha) \left( \frac{\bar{x}}{X} \right) \right]$ Singh and Espejo (2003)</td>
</tr>
<tr>
<td>$k$</td>
<td>1</td>
<td>$t_{kaur} = k \bar{y} \left( \frac{X}{\bar{x}} \right)$ Kaur (1986)</td>
</tr>
<tr>
<td>$k$</td>
<td>0</td>
<td>$t_{kaur} = k \bar{y} \left( \frac{\bar{x}}{X} \right)$ Kaur (1986)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$t_{r} = \bar{y} \left( \frac{\bar{X}}{\bar{x}} \right)$ Cochran (1940)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$t_{p} = \bar{y} \left( \frac{\bar{x}}{X} \right)$ Robson (1967)</td>
</tr>
<tr>
<td>k</td>
<td>α</td>
<td>Estimator</td>
</tr>
<tr>
<td>---</td>
<td>----</td>
<td>----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>k</td>
<td>α</td>
<td>( t_{p2} = k \bar{y} \left[ \alpha \exp \left( \frac{X - \bar{x}}{X + \bar{x}} \right) + (1 - \alpha) \exp \left( \frac{\bar{X} - x}{\bar{x} + X} \right) \right] ) Proposed estimator</td>
</tr>
<tr>
<td>1</td>
<td>α</td>
<td>( t_5 = \bar{y} \left[ \alpha \exp \left( \frac{X - \bar{x}}{X + \bar{x}} \right) + (1 - \alpha) \exp \left( \frac{\bar{X} - x}{\bar{x} + X} \right) \right] ) Singh et al. (2008)</td>
</tr>
<tr>
<td>k</td>
<td>1</td>
<td>( t = k \bar{y} \exp \left( \frac{X - \bar{x}}{X + \bar{x}} \right) )</td>
</tr>
<tr>
<td>k</td>
<td>0</td>
<td>( t = k \bar{y} \exp \left( \frac{\bar{X} - x}{\bar{x} + X} \right) )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>( t_{exp} = \bar{y} \exp \left( \frac{X - \bar{x}}{X + \bar{x}} \right) ) Bahl and Tuteja (1991)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( t_{exp} = \bar{y} \exp \left( \frac{\bar{X} - x}{\bar{x} + X} \right) ) Bahl and Tuteja (1991)</td>
</tr>
</tbody>
</table>